Small businesses and risk contingent credit

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Abstract

Purpose – Firms facing significant income volatility can often suffer from downside risk such that return on assets is insufficient to meet fixed financial obligations. The purpose of this paper is to provide a prescriptive credit solution for small businesses facing exogenous income risk.

Design/methodology/approach – Formulas for risk-contingent operating and collateralized loans are developed and simulated in the context of a specific business sector.

Findings – The paper demonstrates that a structured credit product with an imbedded option can reduce or eliminate financial risks by providing payouts that decrease the amount of principal and/or interest that firms must repay under low income states.

Originality/value – The overall objective of this paper is to provide a means to mitigate exogenous income risk faced by firms through the design and application of a risk-contingent credit product that is tied to primary markets and simple to implement. In this context, risk contingency credit refers to a suite of financial products with payoff schedules (loan principal) that are linked to specific commodities or indices. The authors are in fact unaware of any commercial financial products of the type considered in this paper and thus their approach is a prescriptive solution to the identified problem.

Keywords Small business finance, Risk-contingent credit, Contingent claims, Risk management, Credit

Paper type Research paper

1. Introduction

Firms facing significant income volatility can be affected by downside risk such that return on assets (ROA) is insufficient to meet fixed financial obligations. While large corporations can circumvent this problem by issuing equity instead of taking on debt, small businesses generally do not have this option. Moreover, small businesses often cannot access bond markets to issue contingent claims debt instruments (e.g. commodity-linked bonds). Thus, a flexible credit solution is necessary for any small business facing high income volatility. Given small businesses account for more than half of private-sector employment and gross domestic product (GDP), credit constraints of small businesses are a significant economic issue[1].

There is considerable literature that utilizes contingent claims approaches to credit risk and loan default by linking the payoff structure of a credit instrument to the value of an underlying asset (Schwartz, 1982; Cortazar and Schwartz, 1994; Ingersoll, 1982). For example, Miura and Yamauchi (1998), Atta-Mensah (2004) and O'Hara (1984) all provide models and discussion of commodity-linked bonds in a general context. Boyle and Schwartz (1977), Brennan and Schwartz (1977, 1979), Bacinello and Persson (2002) and Boyle and Tian (2008) discuss and provide models for valuing equity-linked life insurance or equity-linked annuities. Gray *et al.* (2007) show how sovereign debt with risk can be decomposed into options on fundamental assets that are highly correlated

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with sovereign credit risk, providing a structure for risk transfer. There also are some limited applications of agricultural commodity-linked instruments. Applications have been applied to agricultural risks in developing economies with a general focus on sovereign debt (Jin and Turvey, 2002; Turvey, 2006; Priovolis, 1987; Anderson *et al.*, 1989; Myers and Thompson, 1989; Myers, 1992; Claessens and Duncan, 1993; Lu and Neftci, 2008). Related applications to weather and catastrophe bonds include Chantarat *et al.* (2008), Turvey (2008), and Xu *et al.* (2008).

This paper develops a risk-contingent credit instrument that ameliorates exogenous income risk faced by small businesses that do not have the ability to issue equity or bond instruments. We demonstrate that structured credit products with an imbedded contingent claim or option can effectively reduce or eliminate financial risks by providing payouts that decrease the amount of principal and/or interest that the firm must repay under low income states. Moreover, the product can eliminate a significant agency issue that exists when/if small businesses hedge using commodity markets. Formulas for risk-contingent operating and collateralized loans are developed and simulated in the context of a specific sector of the agricultural industry; the dairy industry[2]. However, these products could be utilized by any small business with limited access to equity and/or bond markets and high income volatility which is linked to a commodity or derivative market. A number of non-agricultural small businesses also face commodity and/or weather related income risk (Mattioli and Needleman, 2011).

The overall objective of this paper is to provide a means to mitigate exogenous income risk faced by firms through the design and application of a risk-contingent credit product that is tied to primary markets and simple to implement. In this context, risk contingency credit refers to a suite of financial products with payoff schedules (loan principal) that are linked to specific commodities or indices. We are in fact unaware of any commercial financial products of the type considered in this paper and thus our approach is a prescriptive solution to the identified problem. This paper addresses issues of not only finance scholars but also commercial and agricultural banks, insurance companies, investment banks, and other front line financial practitioners.

2. Business risk, financial risk, and risk-contingent credit

Large corporations can issue equity to by-pass problems associated with high income volatility. However, small businesses generally do not have access to equity or bond markets. While many small businesses are not affected by price volatility, there are a substantial number of small business sectors that face significant income volatility. Specifically, there are many agricultural and non-agricultural small businesses that have commodity and/or weather related business risk (see Table I for specific examples). Hence, numerous business areas could utilize the proposed application of risk-contingent credit.

Table I.
Examples of small business types with hedgeable business risks

RCC hedgeable business risk	Business
Commodity risk (revenue side)	Agricultural industry, mining companies
Commodity risk (expense side)	Transportation companies, construction companies
Weather risk (revenue side)	Hospitality companies, beverage producers, building materials companies
Weather risk (expense side)	Construction companies

Before proceeding to risk-contingent credit, we review conceptually the relationship between business and financial risks[3]. The starting point is the expected return on equity (ROE) and its relationship to the expected ROA and capital structure with weights for assets (A) and debt (D) as a proportion of equity (E):

$$\overline{ROE} = \overline{ROA}\frac{A}{E} - i\frac{D}{E}. \quad (1)$$

Total risk is defined by:

$$\sigma_{ROE} = \sigma_{ROA}\left(1 + \frac{D}{E}\right) \quad (2)$$

and differentiating total risk gives the relationship between business and financial risk:

$$\Delta\sigma_{ROE} = \left(1 + \frac{D}{E}\right)\Delta\sigma_{ROA} + \sigma_{ROA}\Delta\left(\frac{D}{E}\right). \quad (3)$$

The first part of the right hand side in equation (3) captures the change in business risk and this is added to the second term which captures financial risk. Not only does debt leverage higher expected returns in ROE, but also it leverages business risk. In other words, as business risks increase, the variability in ROE will increase linearly with financial leverage. Likewise as debt increases relative to equity, holding business risk constant, total risk will increase accordingly.

Using equation (3) we can define financial risk as the incremental increase in the variability of ROE due to financial leverage. An alternative but equivalent definition of financial risk can be defined by setting equation (1) equal to 0 and solving for the breakeven ROA. We refer to this as the critical ROA:

$$ROA^* = i\frac{D}{A}. \quad (4)$$

The significance of the critical ROA is that it defines the boundary in probability at which fixed financial obligations cannot be met through operating income. We define financial risk as the probability that the ROA will fall below the critical ROA:

$$PROB(ROA < ROA^*) = \int_{l}^{i(D/A)} f(ROA|p,x)dROA. \quad (5)$$

In equation (5) the probability distribution of the ROA is an amalgam of conditional risks tied to price (p) and other random factors (x) that are exogenous to the firm. In this context, financial risk increases as the interest rate and the debt to asset leverage ratio increase[4].

To reduce financial risk as defined by equation (5), businesses can reduce debt, seek lower interest rates, or reduce the tails in $f(ROA|p, x)$. This can be accomplished through a variety of mechanisms including diversification, forward contracting, hedging with futures, or hedging with options. Forward contracts and futures will reduce probabilities in the upper and lower tails while options will reduce probabilities in the lower tails [put (call) option hedge against a price decrease (increase)].

2.1 Additional benefits of risk-contingent credit

In theory, any small business has the option of establishing a traditional hedge to mitigate exposure to risk. However, the proposed type of product has benefits beyond that of a traditional hedge position taken in a derivative market:

- *Lower collateral requirements.* The embedded insurance can act as a substitute for collateral.
- *Decrease in agency issues.* If prices fall, the firm does not have to pay the operating loan in full, yet the lender is guaranteed to receive full payment from the intermediary who deposits indemnities directly into the borrower's credit account at the originating financial institution.
- *Improved credit worthiness.* The effects of market movements that otherwise would have led to arrears, default, or foreclosure are ameliorated due to the elimination of agency issues between the borrower, lender, and intermediary[5].
- *Increase availability of funds for small business loans.* The decreased default risk of small business loans could provide the opportunity for institutions to rebalance their portfolios in favor of small businesses.
- *Decrease in information and transaction costs.* This product eliminates the additional (and often prohibitive for small businesses) information and transaction costs of establishing and unwinding traditional hedge positions.
- *Eliminate margin requirements.* This product would not require margin. Further, the option premia are imbedded into the interest rates, thus eliminating the need for upfront expenditures by the business[6].

3. Risk-contingent credit case study: agriculture – the New York State dairy industry

3.1 Industry background

We use the New York State dairy industry for our case study given that dairy farmers must borrow funds to maintain operations or make new investments under conditions of extreme price uncertainty. The underlying price risk in the dairy industry is from the price of Class III milk, which is traded as futures and options on the CME and is the established base price for milk sold in New York[7]. Thus, our commodity-linked credit is a structured financial product in the form of an operating loan or mortgage with repayment tied to the underlying price of milk. In essence, we attach a "put" option to the credit so that should the price of milk fall below a fixed price, the option component "insures" the loan. If we assume that price is the only source of risk we can define a hedging strategy using options with a payoff equivalent to $Max\ [ROA, ROA^*]$ so that $PROB\ (ROA < ROA^*) = 0$. Figure 1 shows a schematic of this relationship.

To briefly place this in context, consider a dairy farmer who in 2009 received an operating commodity-linked loan with repayment tied to the milk price at maturity. A year later, when the loan matures in 2010, if the milk price rises above the stated (strike) price (contracted when starting the loan), the dairy farmer will repay the full principal amount; however, if milk price declines below the stated price, the dairy farmer will repay the principal minus a prorated difference between stated price and the actual milk price. In this way, the dairy farmer is protected against downside milk prices. As for the financial institution who issued the loan, their payoff does not change because the differences will be paid by the imbedded put option. In essence,

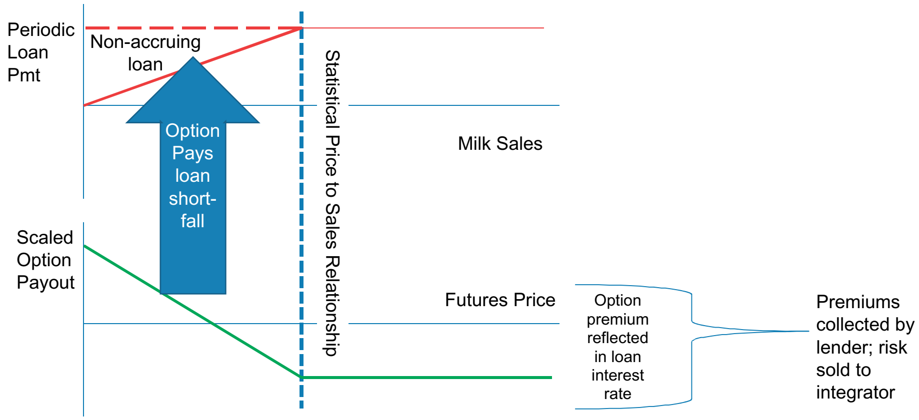


Figure 1.
Schematic of payoff structure of risk-contingent credit

the commodity-linked loan mitigates the financial and business risks of dairy farmers to help them survive economic downturns.

From the perspective of the agricultural banking system this product is also beneficial. As milk prices decline, the dairy farmers' abilities to repay loans decrease; leading to higher default risks. This increased default risk triggers credit rationing and/or increased collateral requirements. Enhanced collateral requirements, be it farmland or essential capital, restrains the farmers' ability to leverage those assets and re-invest in production. Furthermore, since the liabilities held by the dairy farmer are assets held by the lender, and as farmers find it increasingly difficult to meet their fixed financial obligations, the quality of the lender's loan portfolio deteriorates. For example, the September 2010 Annual Report of Farm Credit East reported losses primarily coming from the dairy sector (about 25 percent)[8].

3.2 Theoretical model

Our risk-contingent credit instrument is distinct from the literature that focuses on bond issues or annuities because the risk-contingent credit is issued or originated at the bank level to firms. The price, represented as a risk premium above prevailing market borrowing rates, can be fully hedged and can therefore be priced using simple formulae consistent with equilibrium.

3.2.1 Risk-contingent operating credit. We assume a fixed period operating credit with the farmer borrowing at $t=0$ and repaying at some future time T[9]. Most medium and small farmers require operating loans for a crop year (around eight months to one year).

The present value of such an operating loan for a borrowed amount f is:

$$B_1 = e^{-iT} f e^{(i^{**})T} \quad (6)$$

where i is the bank's discount rate and i^{**} is the interest rate charged to the loan. For a dairy farmer who will plant grains and forage for feed and needs to repay the operating loan from milk sales, the model imbeds into the operating loan contract a commodity option on the price of milk which can be written as:

$$B = e^{-iT}(f e^{i^{*}T} - \Psi[Max(0, Z - P(T))] \quad (7)$$

where Z is the strike price, $P(T)$ is the milk price at time T, i^* is the interest rate charged on the operating loan, and $\Psi = f/Z$ is the hedge ratio. Equations (6) and (7) and rearranging terms gives the formula for the interest rate for risk-contingent operating credit charged by the lender (i^*):

$$i^* = \frac{ln[(\Psi E[Max(0, Z - P(T))]/f) + e^{i^{**}T}]}{T} \tag{8}$$

The term $E[Max(0, Z - P(T))]$ is the payoff value of an option on the tradable futures contract. In this formulation the option is represented as a put option but it can also be a call option if the risk is a price increase. Assuming the price series satisfies a geometric Brownian motion, we can use Black's (1976) formulation for the pricing of options on futures directly by substituting $E[Max(0, Z - P(T))] = ve^{rT}$ into equation (8). Where v is the posted CME options price on a Class III milk futures contract with an expiry at T and r is the risk-free treasury yield used to discount the option[10].

3.2.2 The price for risk-contingent mortgage contracts. The formula for risk-contingent credit is derived similarly. The annuity formula for an N year mortgage of the total loan amount F can be written as:

$$A(i) = F\left[\frac{1-(1+i)^{-N}}{i}\right]^{-1} \tag{9}$$

where $A(i)$ is the amortization of the lump-sum amount F into N smaller cash flows. Now applying the hedge ratio, $\Psi = A(i)/Z$, the value of the mortgage with an embedded commodity option and risk adjusted discount rate r_B is:

$$B = \frac{A(i^*)}{r_B}(1-e^{-r_B N}) - \left(\frac{A(i^*)}{Z}\right)E[Max(0, Z - P(T))]\frac{(1-e^{-r_B N})}{r_B} \tag{10}$$

Equation (10) has a few key imbedded assumptions. First it is assumed that the drift, volatility, and market price of risk of the underlying commodity is, on expectation, constant over the life of the N-year loan. Second if $P_n(0)$ is the beginning-of-period price for year n and $P_n(T)$ is the end-of-period price in year n, then as long as $Z_n = zP_n(0)$ where z is a fixed proportion of the initial price to be covered, then the option cost will be the same in each year.

If r_B is the lender's discount rate applied to the present value of the amortization without any option, then the value of the mortgage to the lender is:

$$B_1 = \frac{A(i)}{r_B}(1-e^{-r_B N}) \tag{11}$$

Now to completely hedge the amortization repayment against commodity output price risk, the mortgage rate (i^*) can be calculated by equating equations (10) and (11) to obtain:

$$\frac{1-(1+i^*)^{-N}}{i^*} = \left[\frac{1-(1+i)^{-N}}{i}\right]\left[1 + \frac{E[Max(0, Z - P(T))]}{Z}\right]^{-1} \tag{12}$$

which can be solved using an iterative process[11].

3.3 Model simulation

3.3.1 Simulation methods. The specific simulation model used is based upon a risk-modified version of the "Dairy Proforma Calculator"[12]. To better represent the realistic situation faced by a typical dairy farm in New York State, the model was modified to include randomness in the monthly milk price received by the dairy farmers. To do this, the contemporaneous (basis) relation between the New York "all-milk price" $P_{NY}(t)$ and the Class III futures (nearby) $P(t)$ over 105 weeks was estimated using ordinary least squares (the *t*-statistics are in parenthesis):

$$P_{NY}(t) = \underset{(8.929)}{3.2582} + \underset{(34.295)}{0.894}P(t) + \varepsilon \sim N(0, 0.706); \quad R^2 = 0.916 \tag{13}$$

The cash price sequence was generated using Monte Carlo techniques to create the futures price series as a random walk[13]:

$$P(t) = P(t-1)e^{((\sigma^2/2)t + (\sigma N(0,1))/\sqrt{t})} \tag{14}$$

Thus, the local "all-milk" price received by farmers is generated using:

$$P_{NY}(t) = 3.2582 + 0.894P(t-1)e^{((\sigma^2/2)t + (\sigma N(0,1))/\sqrt{t})} + N(0, 0.706). \tag{15}$$

The options prices are derived directly using Black's (1976) model for options on futures assuming an at-the-money strike and volatility of 29 percent. Starting in January, options prices were calculated for each month based on the conditions as of January 1, 2010. A one year horizon was assumed for the operating loan so the option applied was calculated for December 2010 (T = 1 year, v = \$1.823/cwt) and the payout based on a \$15.80/cwt strike was calculated from the simulated milk price for December[14]. The mortgage loan has a monthly amortization and included the possibility of independent monthly payments. We used the average value of the option prices across all 12 months, v = \$1.284/cwt, with a payment recorded if in any month the simulated milk futures price fell below the strike of \$15.80/cwt.

We do not consider milk as being the only source of market risk facing the dairy farmer and include feed cost price volatility based on randomness in corn and soybean prices. These also were generated using a correlated random walk with the futures price of milk being the third correlate. The volatilities in the geometric Brownian motion of corn and soybean are calculated from implied volatilities of at-the-money options; using the options calculator from the Chicago Board Options Exchange (CBOE) based on the respective March (2010) contracts. The calculated correlation between soybeans and milk was 0.1033, between corn and soybeans was 0.7369, and between corn and milk was 0.0992.

A third source of risk is the cyclicality in milk production. To add uncertainty to milk production we use the "Pert distribution" which requires only the minimum, maximum and most likely milk production[15]. Historical data of New York State milk production per month from 1970 to 2009 were obtained from the NASS database. Weights for each month were calculated as a percent of total annual production for each year, and from these the minimum, maximum and most likely production weights were derived. This approach not only captures the seasonality in production but also the variability around that seasonality[16].

3.3.2 Simulation results.
3.3.2.1 Risk-contingent operating loan. We first investigate commodity-linkage to an operating loan. The simplest and least costly structure is for a lump-sum loan that a farmer would borrow in full and repay at a prescribed date, say in one year. Operating lines of credit are more difficult because the operating line balance is not known in advance so establishing a pricing mechanism to protect balances when both prices and line balances are moving is very costly and sometimes requires monthly interest rates that would be unacceptable to any firm or lender[17]. While the fixed amount, fixed term, periodic loan is feasible, it is more expensive than an amortized mortgage loan.

We assume in Figure 2 a $100,000 operating loan with a one-year duration and a base interest rate of 6 percent. To imbed an option, the interest rate would have to rise to 16.36 percent. This is higher than an unsecured loan, but lower than most credit card rates. Nonetheless, it has the advantage that if prices fall the farmer does not have to pay the operating loan in full, while the lender receives full payment. In addition, the embedded insurance can act as a substitute for collateral. For example, assuming continuously compounded (geometric) interest rates, the total interest expense, if the prices in December rise above the strike, is $17,776[18]. This is $11,592 higher than the loan without the imbedded option. However, if the price in December falls to, for example, $9.514/cwt the option part of the loan will payout $39,780.12, leaving only $60,219.88 to be paid by the farmer.

Figure 3 shows the distribution of loan repayment and option payment side by side. The symmetry is clear. Our formula calculates an additional interest expense of $11,592. In Figure 3, the left hand distribution has mean indemnities after 10,000 simulations of $11,559, a difference of only 0.285 percent. Across all possible outcomes, the farmer would expect to pay only $88,441 of loan principle. Add to this $17,776 in interest expense and the mean payout on the loan is $106,217. This implies a simulated continuously compounded interest rate of 6.217 percent. We note that with the base operating loan rate of 6 percent $e^{0.06} \rightarrow 6.183$ percent. The key point of interest is

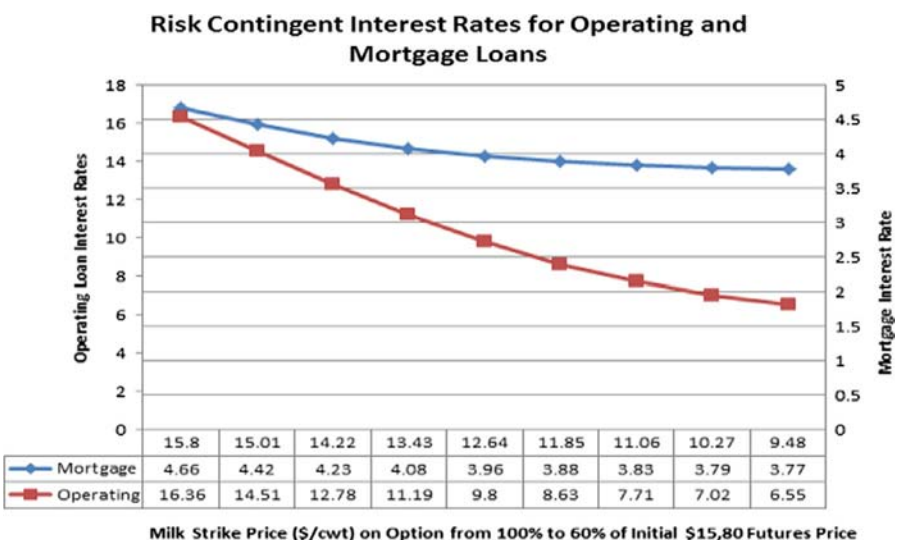
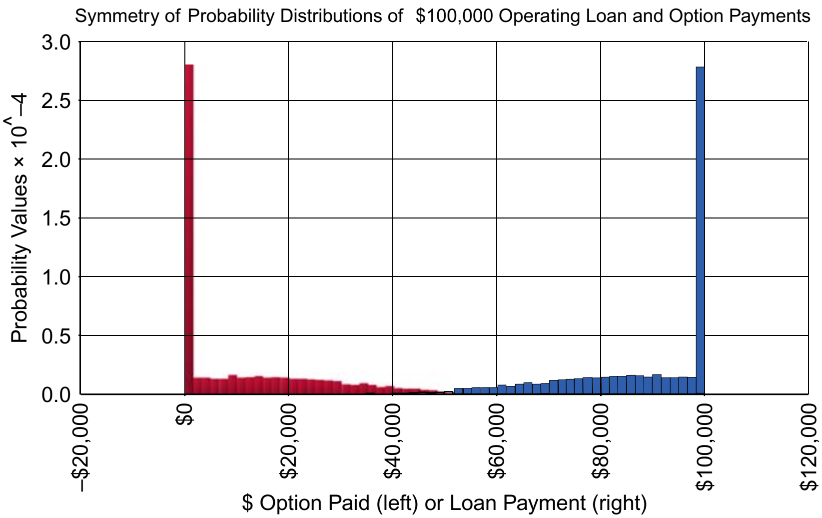


Figure 2.
Risk-contingent interest rates



Option Payment		Loan Paid	
Minimum	$0.00	Minimum	$33,154.53
Maximum	$66,845.47	Maximum	$100,000.00
Mean	$11,458.57	Mean	$88,541.43
Std Dev	$14,275.96	Std Dev	$14,275.96

Figure 3.
Loan repayment and option payment on operating loan

that on average the farmer will be no worse off with a higher interest risk-contingent operating loan over time than a conventional loan, but in times of low prices the farmer is well protected against price risk.

Figure 3 shows Monte Carlo simulated frequencies and magnitudes of payments from the option part (left) linked to Class III milk prices and operating loan payments by the dairy farmer (right).

3.3.2.2 Risk-contingent mortgage loans. Using the simulation model, we also design a mortgage contract with an imbedded put option that would pay the loan down if prices fell below $15.70. We use a base mortgage rate of 3.75 percent on a $192,000, 20 year mortgage for a high risk farmer. The average monthly value of put options on Class III milk as of January 1, 2010 was $1.284. Using these numbers, we calculate that the interest rate on the risk-contingent mortgage would have to increase from 3.75 percent to 4.66 percent to reflect the market risks. This increase is quite small and the monthly mortgage increased from $1,141 to $1,234 as a result. Figure 2 also shows the interest rates for a risk-contingent mortgage as milk strike prices on the imbedded option decrease.

To illustrate how the mechanism works, consider the following random draw for a June payment. From the initial price of $15.80/cwt the futures price declines to $11.42/cwt and the local cash price falls to $13.25/cwt. The option component pays off $316.23 of the total monthly payment of $1,234.03 so that the farmer’s obligation for June is reduced to $917.80.

Figure 4 shows the loan repayment structure for a simulated sequence of milk prices. This graph was selected because the pattern of prices is very similar to what was observed throughout 2010. It can be seen that as the futures price falls below

Figure 4.
Loan repayment with a commodity-linked mortgage

$15.70 the option will be exercised. For example, in month 8 (August) when the price dropped to about $13.75 the farmer would only have had to pay less than 88 percent of that month's mortgage with the remaining 22 percent being paid from the option. Since the option component, by design, is applied directly to the mortgage payment, at no time should the loan fall into arrears; the lender gets paid with virtual certainty and the farmer, facing low prices gets some financial relief. Although the cost of the loan is higher, this is more easily affordable at higher prices.

Figure 5 shows the probability distributions of the payoffs for the first year of the mortgage (based on 10,000 iterations) and the distribution of principal paid by the farmer. Again, the symmetry between what the option pays and what the farmer pays is clear. On average the sum of indemnities and owner contributions almost exactly equals the calculated mortgage payment at 4.66 percent ($14,808/year). From Figure 5, mean option payments are $1,105 while mean farmer payments are $13,704 so that across all possible outcomes the lender receives $14,808. Importantly, this is virtually identical to the calculated mortgage on the base 3.75 percent loan without the risk-contingent credit. Thus, in low price states of nature the farmer receives added liquidity from the imbedded option, while the lender receives full payment regardless of the market price.

The solid line shows a random path of Class III milk prices. The upper shaded area on the bars illustrates how, on a monthly basis, the option part to the mortgage reduces the payment liability to the farmer.

Figure 5 is for a $190,000 mortgage, with 20 year monthly amortization and an annual payment of $14,808 (paid monthly). The distribution on the right is the distribution of options payments and the distribution on the left is the distribution of actual mortgage payments.

The motivation for considering risk-contingent credit is to balance business and financial risk. We provide in Figure 6 the scatter plot between options payouts on the vertical axis and the debt to coverage ratio on the horizontal axis with the critical

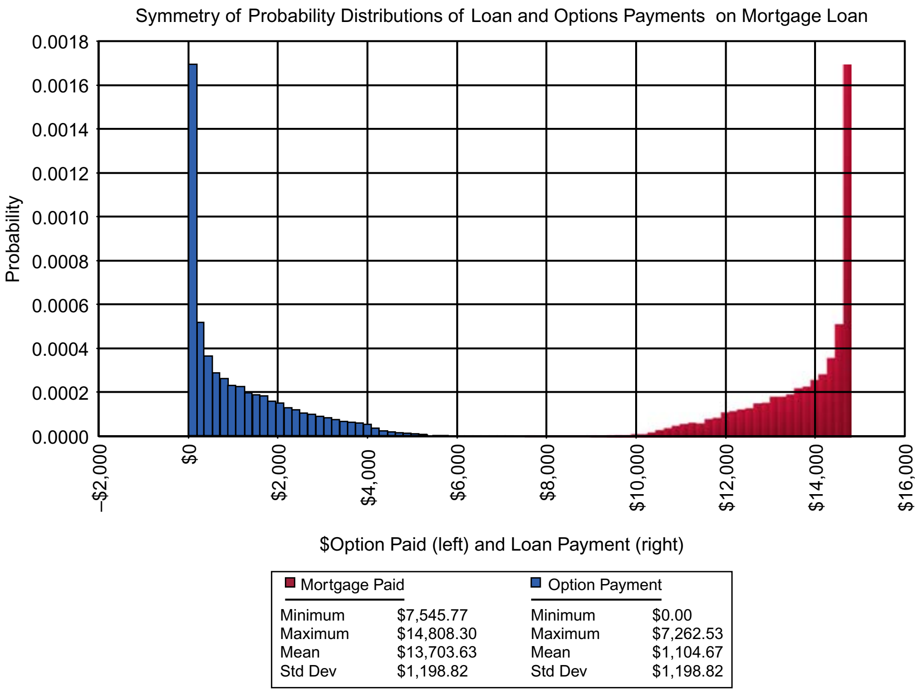


Figure 5.
The probability distributions of loan and options payments

coverage ratio of 1.0 marked. Recall that the scatter diagram is based on outcomes from the farm simulation model which includes not only randomness in prices, but also in raw material costs and production. The figure shows a non-linear relationship with the largest options payments occurring in those states of nature for which the debt coverage ratio (which also includes options payments) approaches or falls below 1. It is precisely in these risky states that the option becomes relevant. The correlation is −0.807. Because the simulation model realistically includes feed cost risk and milk production risk, lack of a perfect correlation is not unexpected.

Figure 6 shows how risk-contingent credit is correlated with low debt repayment capacity. The bulk of payments from the option component occur when the debt servicing ratio nears or falls below 1.0.

4. Concluding remarks

The rising number of small businesses facing financial stress is an important issue that requires new financial products to better meet the needs of businesses while simultaneously reducing the credit risk to lenders. The types of credit products discussed in this paper offer a risk transfer mechanism that could provide significant stability to credit markets for small businesses. Using the New York dairy industry as an example, we have shown how structured financial products can be designed to balance both the business and financial risks. Although these products come with a higher interest rate, their potential to reflect the true market price of risk is beneficial to both

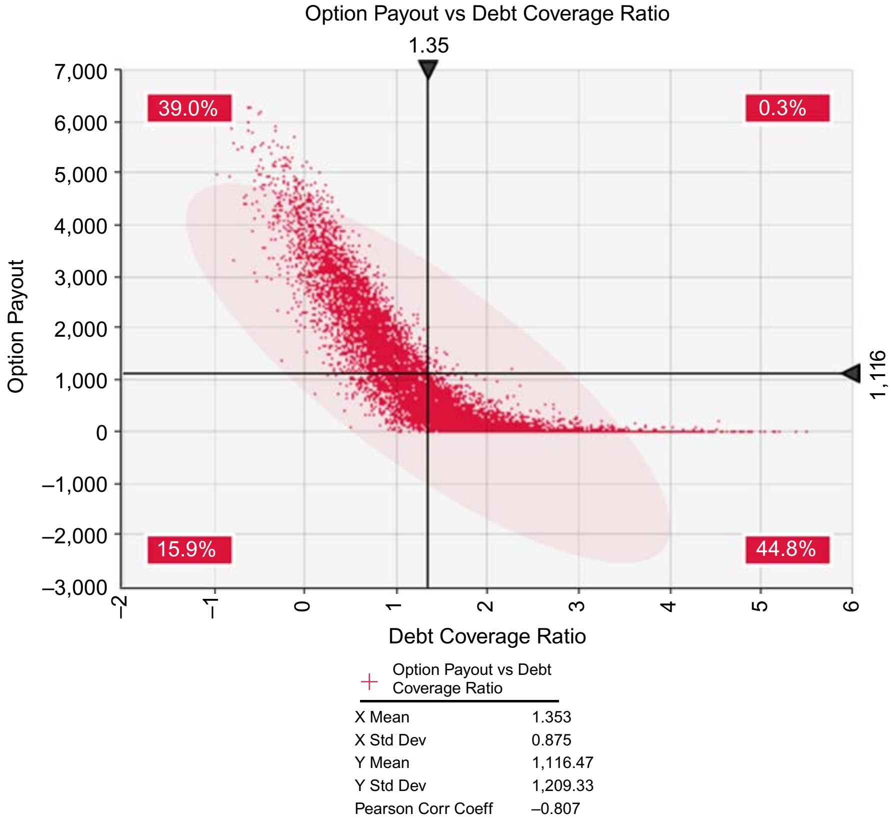


Figure 6.
Relationship between debt coverage ratio and option payout

borrowers and lenders alike. Removing income risk can beneficially improve credit demand since pledged collateral comes with much lower risk of loss. Moreover, lenders can increase supply to these types of borrowers due to the lower credit risk.

For high volatility commodities the risk premium and interest rate would be higher than those reported here. Nonetheless, the increase in marginal rates on operating loans would still be below prevailing credit card rates. In the specific case of the dairy industry, the marginal cost of risk-contingent credit is less than 1 percent higher because of the time value of money and future payments. Nonetheless, we have shown that, across all possible states of nature, the average rate of interest after options indemnities have been paid is precisely the same as the loans without risk transfer.

While small businesses do have other alternatives for risk mitigation, the products discussed have lower information and transaction costs which can be advantageous to small business owners. Furthermore, it has been shown that in certain industries (e.g. agriculture), small business owners that derive a higher percent of income from their small business, hedge less than those who derive a lower percent of income from their small business (Dorfman and Karali, 2010).

There are important institutional considerations with regard to the product described in this paper. In the absence of regulatory changes for deposit taking institutions, this product would require a third party financial institution to originate and underwrite the structured financial product that could then be retailed through commercial lenders. While this third party provider could be either private or public, it would need to be regulated and/or insured by the government. Since small businesses are often the engine driving economic recoveries, this could be a useful government mechanism to spur economic growth.

Notes

1. During the 1998-2004 time period, small businesses produced half of private non-farm GDP (Kobe (2007)).
2. For example, dairy farmers in New York State face significant volatility in the prices they receive for milk production. In 2009 dairy farmers faced one of the worst low price situations in history. Dairy farm net income was squeezed and many farmers were unable to meet their debt obligation. With costs of production approaching nearly $16 per hundred pounds of milk, the rapid decline in prices throughout 2009 and into 2010 combined with rapid rises in feed costs created conditions unseen since the 1970s. According to the New York State Department of Agriculture & Markets, milk is New York's leading agricultural product with milk sales account for one-half of total agricultural receipts. As the third leading producer nationwide, production in 2008 was 12.4 billion pounds with a value of $2.3 billion. Thus, assisting existing and prospective dairy farmers to manage their risks would greatly contribute to stabilizing farmer income.
3. By financial risk we refer to the possibility that a business's cash flows are insufficient to pay creditors and fulfill other financial responsibilities. The level of financial risk relates to the amount of debt a business incurs to finance operations. The more debt a business has, the more likely it is to default on its financial obligations. By business risk we refer to the possibility that a business's cash flows are insufficient to cover its operating expenses like the cost of goods sold, rent, and wages. Unlike financial risk, business risk is independent of the amount of debt a business has.
4. Other factors can contribute to output uncertainty. For the dairy industry example presented in this paper, production is relatively constant and controllable. For other agricultural crops, output can be uncertain but can also be hedged through the use of crop insurance, offered in the US through the Risk Management Agency. However, in the formulas that follow, the risk-contingent credit can be applied to any source of risk including production and weather. Typically, however, joint risk products are not tradeable and are highly complex structures. In this paper, it is assumed that price contributes most to total risk and we use an example of a commodity that is traded on the Chicago Mercantile Exchange (CME).
5. The interlinked contract does not eliminate all risks. There are other factors (which may be insurable) that still could lead to default on the loan.
6. The intermediary required to purchase the risk could be a large hedge fund, investment bank, or government sponsored enterprise (GSE). Since the intermediary would be writing an option, it would be required to post margin.
7. During the ten years period from 2000 to 2010, "all-milk prices" received by New York State dairy farmers were extremely volatile. (According to California Department of Food and Agriculture, the all milk price is defined as a weighted average of the prices dairy processors pay for all grades A and B milk, calculated by the National Agricultural Statistics Service (NASS) and usually reported for milk of average fat test.) The mean of all milk prices was $15.25/cwt (hundred weight) and the standard deviation was $2.83/cwt during the 120 month period; the highest price was $22.8/cwt, reached in September 2007 but 19 months later the price hit a low of $11.2/cwt in March 2009. (For a typical New York dairy farm with 50 cows producing about 850 cwt of milk per month, the reduction in monthly cash flow would have fallen from $19,380 to $9,520). The Dairy Business Summary of New York State 2009

compared with that of 2008 showed that 60 percent of the 41 New York dairy farms with 100 or more cows surveyed generated negative net income (adjusted for inflation) in 2009, with the 20 percent worst hit farm losing on average $781,694. In 2009, the average net farm income without appreciation was −$144,714, a sharp decline from 2008, when the same 41 dairy farms generate an average of $544,151 of net income. Decreasing profitability also adversely affected the solvency of these farms, with the debt coverage ratio declining from 2.00 in 2008 to 0.21 in 2009. This means that in 2009, for every dollar of scheduled repayments, dairy farmers, through the earnings of their businesses, were $0.79 short.

8. www.farmcrediteast.com/About-Us/Inside-Farm-Credit-East/~/media/092010Quarterly.ashx
9. This model does not work for operating lines of credit or other forms of credit with variable loan balances.
10. To illustrate how equation (8) works, assume a $100,000 operating loan. The January futures price for milk is $15.80/cwt with volatility of 0.29. An at the money put option is priced using Black's model with an expiry in December and treasury yield of 1 percent. The current fixed rate on the operating loan for one year is 6 percent. The hedge ratio is $\Psi = (f/Z) = (10{,}000/15.75) = 632.91$ and from equation (8) the interest rate on the operating loan is:

$$t^* = \frac{ln[(639.27 \times 1.27e^{0.01}/10,000) + e^{0.06}]}{1} = 0.134.$$

In words, to cover a $10,000 operating loan, an equivalent of 639.27 options with an options price of 1.276 will result in an interest rate of 13.4 percent, or a 7.4 percent risk premium for protection on the loan if the futures price falls below $15.8/cwt.

11. As an example, suppose the farmer wants to raise $100,000 for investment in dairy by mortgaging the property for 20 years. The monthly amortization for a 20-year mortgage at 3.75 percent base interest rate is $592.89 per month. The embedded at-the-money option as calculated previously is 1.2805. Solving equation (12) we get i = 4.66 percent which is only 0.91 percent higher than the base interest rate of 3.75 percent and when applied to the farm mortgage, yields an amortization of $641.05 per month. If the price of milk at the end of each month falls below $15.80/cwt the option pays off a portion of the loan accordingly.
12. Center for Dairy Profitability, University of Wisconsin-Madison (http://cdp.wisc.edu/Decision%20Making%20Tools.htm).
13. Monte Carlo techniques from @RISK www.palisade.com
14. cwt: hundred weight – a unit of weight measurement that is equal to exactly 100 pounds.
15. @RISK www.palisade.com
16. For space considerations, we do not include the full calculations. However, to illustrate, we calculate that the lowest production month on average was February with 7.72 percent of annual production with a minimum of 7.40 percent and maximum of 8.17 percent, while the highest production month on average was May with 9.10 percent of annual production with a high of 9.80 percent and a low of 8.75 percent. December was the buffer month so that for each iteration the weights summed to 1.0.
17. This is not to say that it is infeasible. Our formula requires that the hedge ratio is tied to the outstanding line of credit balance which could be any value depending on risk. In any month the hedge ratio needs to be rescaled to that month's beginning line of credit balance using the beginning of year strike price. If the line of credit increased due to a price decrease, the option component would be in-the-money such that the risk premium and interest rates would be extremely high. It was not uncommon in our simulations to find marginal interest rates in excess of 250 percent. Since in large part our focus was on simple solutions we concluded

that applying risk-contingent credit to operating lines of credit was not practical and therefore leave out further discussion.

18. There are variants on this that may be more attractive. For example, the 16.36 percent rate is based upon the option value in December ($1.823/cwt), 12 months after the loan was taken. However, it may be helpful to use an average option price which captures inter-temporal risks as they might occur throughout the year. Using the average options value $1.2843 and basing the payout on the average monthly price, the interest rate is 13.41 percent. For example, if the average monthly price falls to $13.548/cwt the option payout will be $14,252 so that the farmer will only repay $85,748 of the $100,000 operating loan.

References

Anderson, R.W., Gilbert, C.L. and Powell, A. (1989), "Securitization and commodity contingency in international lending", *American Journal of Agricultural Economics*, Vol. 71 No. 2, pp. 523-30.

Atta-Mensah, J. (2004), "Commodity-linked bonds: a potential means for less-developed countries to raise foreign capital", Working Paper 2004-20, Bank of Canada, June.

Bacinello, A.R. and Persson, S. (2002), "Design and pricing of equity-linked life insurance under stochastic interest rates", *Journal of Risk Finance*, Vol. 3 No. 2, pp. 6-21.

Black, F. (1976), "The pricing of commodity contracts", *The Journal of Financial Economics*, Vol. 3 Nos 1/2, pp. 167-79.

Boyle, P. and Schwartz, E.S. (1977), "Equilibrium prices of guarantees under equity-linked contracts", *The Journal of Risk and Insurance*, Vol. 44 No. 4, pp. 639-60.

Boyle, P. and Tian, W. (2008), "The design of equity-indexed annuities", *Insurance: Mathematics and Economics*, Vol. 43 No. 3, pp. 303-15.

Brennan, M.J. and Schwartz, E. (1977), "Convertible bonds: valuation and optimal strategies for call and conversion", *The Journal of Finance*, Vol. 32 No. 5, pp. 1699-715.

Brennan, M.J. and Schwartz, E. (1979), "Alternative investment strategies for the issuers of equity linked life insurance policies with an asset value guarantee", *The Journal of Business*, Vol. 52 No. 1, pp. 63-93.

Chantarat, S., Turvey, C.G., Mude, A.G. and Barrett, C.B. (2008), "Improving humanitarian response to slow-onset disasters using famine-indexed weather derivatives", *Agricultural Finance Review*, Vol. 68 No. 1, pp. 169-95.

Claessens, S. and Duncan, R.C. (1993), *Managing Commodity Price Risk in Developing Countries*, The Johns Hopkins University Press, Baltimore, MD.

Cortazar, G. and Schwartz, E. (1994), "The valuation of commodity contingent claims", *Journal of Derivatives*, Vol. 1 No. 4, pp. 27-39.

Dorfman, J.H. and Karali, B. (2010), "Do farmers hedge optimally or by habit? A Bayesian partial-adjustment model of farmer hedging", *Journal of Agriculture and Applied Economics*, Vol. 42 No. 4, pp. 791-803.

Gray, D.F., Merton, R.C. and Bodie, Z. (2007), "Contingent claims approach to measuring and managing sovereign credit risk", *Journal of Investment Management*, Vol. 5 No. 4, pp. 5-28.

Ingersoll, J.E. (1982), "The pricing of commodity-linked bonds: discussion", *The Journal of Finance*, Vol. 37 No. 2, pp. 540-1.

Jin, Y. and Turvey, C.G. (2002), "Hedging financial and business risks in agriculture with commodity-linked loans", *Agricultural Finance Review*, Vol. 62 No. 1, pp. 41-57.

Kobe, K. (2007), "The small business share of GDP, 1998-2004", Small Business Research Summary, No. 299, Small Business Association Office of Advocacy.

Lu, Y. and Neftci, S.N. (2008), "Financial instruments to hedge commodity price risk for developing countries", IMF Working Papers, January.

Mattioli, D. and Needleman, S. (2011), "For small businesses, recession is not over", *The Wall Street Journal*, 6 July.

Miura, R. and Yamauchi, H. (1998), "The pricing formula for commodity-linked bonds with stochastic convenience yields and default risk", *Asia-Pacific Financial Markets*, Vol. 5 No. 2, pp. 129-58.

Myers, R.J. (1992), "Incomplete markets and commodity-linked finance in developing countries", *World Bank Research Observer*, Vol. 79 No. 12, pp. 79-94.

Myers, R.J. and Thompson, S.R. (1989), "Optimal portfolios of external debt in developing countries: the potential role of commodity-linked bonds", *American Journal of Agricultural Economics*, Vol. 71 No. 2, pp. 517-22.

O'Hara, M. (1984), "Commodity bonds and consumption risks", *The Journal of Finance*, Vol. 39 No. 1, pp. 193-206.

Priovolis, T. (1987), "Commodity bonds: a risk management instrument for developing countries", Working Paper No. 1987-12, World Bank, Washington, DC, November.

Schwartz, E.S. (1982), "The pricing of commodity-linked bonds", *The Journal of Finance*, Vol. 37 No. 2, pp. 525-39.

Turvey, C.G. (2006), "Managing food industry business and financial risks with commodity-linked credit instruments", *Agribusiness: An International Journal*, Vol. 22 No. 4, pp. 523-45.

Turvey, C.G. (2008), "The pricing, structure, and function of weather-linked bonds, mortgages, and operating credit", *Agricultural Finance Review*, Vol. 68 No. 1, pp. 135-50.

Xu, W., Odening, M. and Muhoff, O. (2008), "Optimal design of weather bonds", paper presented at the Annual Meeting of the American Agricultural Economics Association (AAEA), Orlando, FL, July 27-29,.

Further reading

Black, F. and Scholes, M. (1973), "The pricing of options and corporate liabilities", *Journal of Political Economy*, Vol. 81 No. 3, pp. 637-59.

Cox, J.C. and Ross, S.A. (1976), "The valuation of option for alternative stochastic processes", *The Journal of Financial Economics*, Vol. 3 Nos 1/2, pp. 145-66.

Heath, D., Jarrow, R. and Morton, A. (1992), "Bond pricing and the term structure of interest rates: a new methodology for contingent claims valuation", *Econometrica*, Vol. 60 No. 1, pp. 77-105.

Hull, J.C. (2006), *Options, Futures, and Other Derivatives*, Prentice-Hall, Upper Saddle River, NJ.

Petzel, T.E. (1989), "Financial risk management needs of developing countries: discussion", *American Journal of Agricultural Economics*, Vol. 72 No. 2, pp. 523-30.

Varangis, P. and Larson, D. (1996), "Dealing with commodity price uncertainty", Policy Research Working Paper No. 1667, World Bank, Washington, DC.