License to Spend: Consumption-Income Sensitivity and Portfolio Choice

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JEL classification: D11, D12, D14, G11.

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Abstract – Contrary to the predictions of traditional life-cycle models, households do not engage in perfect consumption smoothing. Instead, consumption tracks current income. Similarly, weak evidence of income hedging runs against standard portfolio theory. We link these two puzzles by proposing a model in which current income is an entitlement to consume, or a license to spend. License-to-spend investors feel more entitled to consume as income rises and do not perfectly smooth consumption. Therefore, they are also less interested in the income hedging potential of financial assets. We test the license-to-spend model using data from the Panel Study of Income Dynamics and find that households whose consumption tracks current income also exhibit a weakened income hedging motive in their portfolio decisions. Overall, we show that the absence of income hedging is the portfolio choice analogue of imperfect consumption smoothing.

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I. Introduction

Consumption and portfolio decisions are fundamentally interrelated because they are governed by the same preferences (e.g., Merton (1969), Samuelson (1969)). However, the empirical literatures on consumption and portfolio choice have developed in relative isolation. A common conclusion in both strands of literature is that standard economic models cannot fully explain household decisions. Traditional life-cycle models predict that household consumption depends on life-time income. Yet, empirical evidence documents that household consumption is excessively sensitive to current income.\(^1\) Canonical portfolio choice models suggest that households should engage in income hedging when making portfolio decisions because income risk cannot be traded or insured.\(^2\) Nevertheless, existing studies have not detected a strong income hedging motive in portfolio decisions.\(^3\)

In this paper, we bridge the gap between these two literatures, and identify a novel connection between consumption smoothing and portfolio decisions. Our key conjecture is that households who do not smooth consumption (i.e., exhibit excess sensitivity of consumption to current income) might be less concerned about income hedging in their portfolio decisions. First, we formalize this conjecture and propose a theoretical framework in which current income is an entitlement to consume, or a license to spend. We show that license-to-spend investors do not engage in perfect consumption smoothing. As a consequence, they are less interested in the income hedging potential of financial assets. Second, we test the license-to-spend model using data from the Panel Study of Income Dynamics. We find that investors who do not engage in perfect consumption smoothing also exhibit a weakened income hedging motive in their portfolio decisions. To our knowledge, we are the first to jointly explore the implications of consumption-income


We begin our analysis with a model that can generate both weak consumption smoothing and weak income hedging. The novel feature of the model is that individuals treat income as an entitlement to consume, or as a license to spend. This feature is inspired by Akerlof (2007). Based on evidence from sociology and behavioral economics, Akerlof (2007) argues that household consumption is affected by consumption entitlements, with current income being the primary determinant of such entitlements.\footnote{For example, see Keynes (1936), Weber (1958), Tversky and Kahneman (1981), Bourdieu (1984), Shefrin and Thaler (1988), and Guiso et al. (2006).} We model the entitlement effect by including current income in the utility function so that each unit of consumption becomes more desirable when current income rises. We then embed the license-to-spend preferences in a consumption/portfolio choice model with exogenous labor supply (e.g., Viceira (2001), Campbell and Viceira (2002)). We solve the model analytically, and show that the license-to-spend effect results in a lack of perfect consumption smoothing. Specifically, a strong license-to-spend effect leads to a positive correlation between optimal consumption growth and current income growth.

We also find that in the license-to-spend model consumption smoothing is related to portfolio decisions. Specifically, in contrast to canonical portfolio choice models (e.g., Viceira (2001) and Campbell and Viceira (2002)), the optimal equity share in the model is directly affected by whether investors have a preference for consumption smoothing or not. The license-to-spend model essentially predicts that a high consumption-income sensitivity weakens the income hedging motive. The attenuation of income hedging arises naturally because investors who are not concerned with smoothing consumption do not value the relative income hedging properties of financial assets. The weakening of the income hedging motive due to consumption-income sensitivities is a novel feature of the license-to-spend model that has not been studied in the literature.

Next, we use household data from the Panel Study of Income Dynamics (PSID) and empirically test the predictions of the model. We use the PSID because it is the only lon-
gitudinal survey of U.S. households that includes data on both consumption and portfolio decisions. In our empirical exercise, we first confirm that, on average, PSID households do not smooth consumption by showing that their consumption growth tracks current labor income growth. We find that a one standard deviation increase in labor income growth leads to about a 2.6% increase in consumption growth. Interestingly, we find that consumption-income sensitivity is strong even among the wealthiest households in the sample. These households do not face borrowing constraints that can potentially explain consumption-income sensitivity (e.g., Jappelli (1990), Runkle (1991), Parker (2014)).

In our portfolio choice tests, we examine how preferences for consumption smoothing affect the portfolio decisions of PSID households. To implement the portfolio tests, we first estimate household-level regressions of consumption growth on income growth. We use the coefficient estimates on income growth to measure the consumption-income sensitivity for each household. We interpret these consumption-income sensitivities as an index of consumption smoothing, where the higher the sensitivity, the lower the desire to smooth consumption.

We use the household-level sensitivities to construct an interaction term that measures the attenuation of income hedging. The license-to-spend model suggests that consumption-income sensitivity affects portfolio decisions through an interaction with the traditional income hedging motive. A standard proxy for the traditional income hedging motive is the correlation between household income growth and stock market returns (e.g., Vissing-Jørgensen (2002b), Massa and Simonov (2006)). Therefore, we interact consumption-income sensitivities with income-growth market-return correlations for each household. This interaction term is our income hedging attenuation measure and it is the main explanatory variable in our market participation (Probit) and asset allocation (Tobit and Heckman) regressions.

Consistent with model predictions, we find that the attenuation of income hedging is an important determinant of portfolio decisions. Specifically, the interaction term measuring the attenuation effect is economically and statistically significant in our portfolio
choice regressions. For example, our estimates from the Tobit regressions suggest that a one standard deviation increase in the income-hedging attenuation term leads to a 3.0% increase in the portion of wealth allocated to risky assets. This effect is comparable to the impact of income risk, which is one of the most important determinants of equity allocation: a one standard deviation decrease in the volatility of income growth leads to a 4.0% increase in the equity share. Further, the attenuation effect is stronger than the traditional income hedging motive: a one standard deviation increase in the income-growth market-return correlation leads to just a 0.6% decrease in the equity share.

We continue to find a strong attenuation effect even when we focus on market participants alone and estimate Heckman (1979) regressions. The Heckman estimates suggest that a one standard deviation increase in the income hedging attenuation term leads to a 1.3% increase in the equity share. This effect is comparable to the impact of wealth, which is also an important determinant of equity allocation: a one standard deviation increase in wealth leads to a 2.2% increase in the equity share. Overall, we find that the income hedging motive is attenuated for PSID households with high consumption-income sensitivity.

A potential concern with our empirical results is that the household-level consumption-income sensitivities are estimated quantities, and may introduce generated regressor biases into our tests. To address this issue, we conduct a bootstrap simulation exercise, and obtain standard errors that account for potential estimation noise in the consumption-income sensitivities. We find that the attenuation of the income hedging motive remains statistically significant even when we use the bootstrapped standard errors. Further, the bootstrap simulations suggest that potential generated regressor biases in our estimates are economically and statistically insignificant. Taken together, these results suggest that our empirical results are robust to measurement error.

We close our empirical analysis with a series of robustness tests. First, we address the potential concern that the income-hedging attenuation term simply captures the non-linear effects of income and wealth on portfolio decisions. We also rule out the
possibility that the attenuation term is capturing risk aversion. Since the PSID does not have a direct measure of risk aversion, we proxy for risk aversion using the volatility of household income growth. This is a reasonable risk proxy because according to Ranish (2013) argues, labor choices are endogenous and more risk seeking investors will self-select into riskier occupations. We find that the attenuation of income hedging is strong in the presence of the income risk proxy.

Next, we examine the impact of borrowing constraints. We expect that the presence of borrowing constraints is not affecting our results because borrowing constraints should have an opposite effect on portfolios decisions than the license-to-spend effect. In particular, in periods when income is low and borrowing constraints might be severe, the investor will probably decrease its optimal portfolio weight to finance consumption. Therefore, if the estimated consumption income sensitivities capture borrowing constraints, they should be negatively related to the optimal portfolio weight. In contrast, the license-to-spend effect operates through the hedging motive and it has a positive effect on the optimal portfolio weight. Therefore, to account for borrowing constraints, we add the estimated consumption income sensitivities as a separate control in our regressions. We find that consistent with our expectations, the attenuation effect remains positive and strong in the presence of proxy for borrowing constraints.

Finally, the license-to-spend story might be related to habit formation models (e.g., Campbell and Cochrane (1999), Korniotis (2008), Gomez, Priestley, and Zapatero (2009)). The connection with habit formation models comes from the fact that individual income might be a proxy for the consumption of the household reference group. To account for aggregate peer effects, as in the habit model of Campbell and Cochrane (1999), we include year fixed effects in the Probit, Tobit and Heckman regressions. Unfortunately, the PSID does not have enough information to allow us to construct peer groups for each household and compute household-level habit measures. Instead, we introduce a series of fixed effects in our regressions that can potentially account for peer effects; they are occupation fixed effects, industry fixed effects, and regional fixed effects based on
the location of the households. The introduction of these fixed effects does not affect the significance of the income-hedging attenuation term suggesting that we are probably capturing something different than habit formation.

Overall, our findings are related to the household finance literature, where the evidence for income hedging has been mixed. On the one hand, Heaton and Lucas (2000) find weak evidence in support of the income hedging motive in the investment decisions of entrepreneurs. Vissing-Jørgensen (2002b) finds no evidence that the correlation between income growth and market returns influences portfolio decisions. Further, Massa and Simonov (2006) show that income hedging motives do not influence the portfolio decisions of Swedish investors. On the other hand, Bonaparte et al. (2013) find that Dutch and U.S. households consider the comovement between income growth and market returns when making portfolio decisions. A common feature of these studies is examining portfolio decisions in isolation from consumption decisions. Instead, we examine both decisions jointly, and show that consumption-income sensitivity attenuates the income hedging motive.

Our work is also related to the literature on the excess sensitivity of consumption to current income. A leading explanation of consumption-income sensitivity is borrowing constraints.\textsuperscript{5} Borrowing restrictions are a reasonable explanation, especially for young individuals who have not accumulated a substantial stock of wealth. However, we find that even wealthy households in our sample choose not to smooth consumption. Another interesting feature of the license-to-spend model is that it can account for consumption-income sensitivity when changes in income are predictable. However, even recent state-of-the-art models with borrowing constraints, permanent and transitory labor income shocks, and hyperbolic discounting\textsuperscript{6} cannot explain the responsiveness of consumption to anticipated income fluctuations.\textsuperscript{7}


The rest of the paper is organized as follows. Section II presents the license-to-spend model. Section III describes the PSID data. Section IV presents evidence that consumers in the PSID do not smooth consumption. Section V examines how a lack of consumption smoothing affects the portfolio decisions of PSID households. Section VI concludes with a brief discussion.

II. Income as Consumption Entitlement

In this section we present the license-to-spend model. The model is an extension of the dynamic portfolio choice model with fixed labor supply (i.e., Viceira (2001)). The license-to-spend model does not include features like borrowing constraints or market participation costs that would hinder us from solving the model analytically. We choose to work with a simple tractable model, so that we can obtain analytical solutions for the optimal portfolio weights and clearly illustrate how the absence of perfect consumption smoothing affects portfolio decisions.\textsuperscript{8}

II.A. Utility from Consumption and Income

We assume that income, $Y_t$, is an entitlement to consume or a license to spend. To make current income a consumption entitlement, we assume that consumption, $C_t$, becomes more desirable as current income increases. Formally, we assume that the marginal utility of consumption is increasing in current income. In the model, we make the marginal utility dependent on income by including income in the utility function:

$$U(C_t; Y_t) = \frac{C_t^{1-\gamma}}{1 - \gamma} Y_t^\theta.$$  \textsuperscript{(1)}

\textsuperscript{8}Generally, there are no closed-form solutions for the optimal portfolio weights in multiperiod models. To gain tractability Campbell and Viceira (1999) and Jurek and Viceira (2011) assume that log returns are predictable and follow an autoregressive model with normally distributed disturbances. Brennan and Xia (2002) use numerical methods to solve the dynamic portfolio problem assuming that returns are predictable due to asset pricing anomalies. Wachtler (2002) examines portfolio choice under complete markets and mean-reverting returns. Chacko and Viceira (2005) assume that expected return are constant across time but return volatility is time-varying. Liu (2007) assumes that returns are quadratic following general Markovian processes.
The license-to-spend utility function is an extension of the constant relative risk aversion (CRRA) model. The constant $\gamma$ is the risk aversion (RA) parameter, and the coefficient $\theta$ determines the importance of the license-to-spend effect.\footnote{We require that $\gamma > 1$ so that the elasticity of intertemporal substitution $(1/\gamma)$ is lower than one. For $\gamma > 1$, we also require that $0 \leq \theta < (\gamma - 1)(1 - \phi_1)$ in which $(1 - \phi_1)$ is the income elasticity of consumption. Since the consumption function is an isoelastic aggregator of income and wealth ($C_t = e^{\phi_0}W_t^{\phi_0}Y_t^{1-\phi_1}$), these conditions on $\theta$ guarantee that the utility function is increasing in income and concave.} For positive $\theta$, an extra dollar of consumption becomes more valuable if it is financed by an increase in current income. In other words, individuals feel happier when they finance their consumption from current income rather than tapping into their savings or taking on debt. For a detailed description of the properties of the utility function in (1), see Appendix A.

The assumption that income is an entitlement to consumption is consistent with the sociology literature. For instance, according to Akerlof (2007), there is ample evidence that consumption decisions are affected by norms about what people think they are entitled to consume.\footnote{For example, see Tversky and Kahneman (1981), Bourdieu (1984), Shefrin and Thaler (1988), and Guiso et al. (2006).} He argues that

“First, sociology gives motivations for consumption that are very different from the reasons for it in the life-cycle hypothesis. A major determinant of consumption is what people think they should consume. Second, what people think they should consume can often be viewed either as an entitlement or as obligations. Finally, in turn, current income is one of the major determinants of these entitlements, and obligations.” (Akerlof (2007), p. 15).

Inspired by Akerlof’s arguments, we include current income in the utility function so that as income rises, marginal utility also rises, and consumers feel more entitled to increase their consumption spending.

Several reasons guide our choice to include current income in the utility function instead of life-time income or wealth. First, based on Akerlof’s (2007) arguments, life-time income does not affect consumption entitlements, which are mainly driven by current income. Second, including income in the utility function allows us to solve analytically
for the consumption function and optimal portfolio weights, which is not possible with wealth in the utility function. More importantly, labor income seems to be a more plausible proxy for consumption entitlements than wealth. This is because consumption entitlements essentially reflect internal and external influences on the proper level of consumption. Labor income can capture internal consumption entitlements better than wealth because labor income represents compensation for the provision of effort. In other words, labor income, as opposed to wealth, can capture preferences related to a “work hard, play hard” mentality, which is related to the internal license to spend. Labor income can also capture external consumption entitlements better than wealth since individuals are usually better informed about the labor income of their peers than they are about the wealth of their peers.

Including income in the utility function is also consistent with evidence from behavioral economics. To better illustrate the link with the behavioral literature, we can rewrite the utility function by replacing $Y_t/C_t$ with $1/(1-s_{y,t})$, where $s_{y,t} = (Y_t - C_t)/Y_t$ is the savings rate out of income:

$$U(C_t; Y_t) = C_t^{1-\gamma+\theta} \left( \frac{1}{1-s_{y,t}} \right)^\theta.$$  

This alternative specification of the license-to-spend model suggests that investors derive utility from consumption as well as savings. This is consistent with findings that consumers view savings as a separate decision, instead of simply a residual to consumption (e.g., Furnham and Argyle (1998)).

A utility function defined over consumption as well as savings is also consistent with the debt aversion model of Prelec and Loewenstein (1998). They argue that the process of spending involves an immediate pain of paying that can reduce the pleasure of consuming. Thus, the utility process should be the sum of the happiness from consuming and the grief from paying instead of saving. The disutility of paying is the highest when spending is financed by borrowing.
Thaler (1985) also proposes a transaction utility theory in which transactions involve both acquisition utility and transaction utility. Thaler’s analysis is similar to the debt aversion framework of Prelec and Loewenstein (1998). Both papers stress that the process of buying a good has two dimensions: acquisition and transaction. In our framework, the transaction utility is related to the reward from saving.

Finally, adding income in the utility functions is also consistent with the work of Duesenberry (1949). Duesenberry, and more recently Akerlof (1997) and Ljungqvist and Uhlig (2000) argue that individuals compare themselves with others and specifically they compare their income with their peers. In other words, they feel happier if their income relative to their peer group is high. However, we do not follow the relative income approach because the peer group of each household is unobservable.

II.B. Life-Cycle Consumption-Portfolio Model

We embed the license-to-spend effect in the dynamic portfolio choice model of Viceira (2001) in which investors have access to a risky and a risk-free asset. In our model, investors can either be employed or retired. When investors are employed, they receive a non-tradeable endowment $Y_t$ (labor income). Retirement can occur with probability $\pi_r > 0$, while $\pi_e = 1 - \pi_r$ is the probability of staying employed. Retirement is an absorbing state and is independent of income growth shocks or asset returns. During retirement, investors receive a constant pension $\bar{Y}$, which is equal to the last pre-retirement income payment.

To close the model, we follow Viceira (2001) and assume that income growth during employment is an i.i.d. process with constant volatility given by

$$\Delta y_{t+1} = \mu_y + \sigma \Delta \epsilon_{y,t+1},$$

where $\epsilon_{y,t+1}$ are i.i.d. $N(0,1)$ random variables. We also assume that the risk-free rate is constant, and that the log return on the risky asset $r_m$ is normally distributed with
constant mean and volatility:

\[ r_{m,t+1} = \mu_m + \sigma_m \epsilon_{m,t+1}, \]

where \( \epsilon_{m,t+1} \) are i.i.d. \( N(0,1) \) shocks. Finally, the correlation between income growth shocks \( (\epsilon_y) \) and asset return shocks \( (\epsilon_m) \) is \( \rho_{y,m} \).

Next, we derive the optimal consumption and portfolio rules. We first consider the investor’s decisions post-retirement. Then, conditional on the post-retirement decisions, we use backward induction to solve for optimal consumption and portfolio rules pre-retirement. The behavior of the investor prior to retirement is the basis for our empirical analysis.

II.C. Consumption and Portfolio Rules During Retirement

During retirement, the investor chooses consumption and portfolio weights to maximize his lifetime utility by solving the following maximization problem:

\[
\max_{\{C_t\}_{t=0}^{\infty}, \{a_t\}_{t=0}^{\infty}} E_t \left[ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{C_{\tau}^{1-\gamma}}{1-\gamma} \bar{Y}^{\gamma} \right], \text{ subject to } \quad W_{t+1} = (W_t - C_t + \bar{Y}) [a_t (e^{r_{m,t+1}} - e^{r_{f,t+1}}) + e^{r_{f,t+1}}] \forall t, \tag{3}
\]

where \( \beta \in (0,1) \) is the rate of time preference, \( W_t \) is wealth, and \( a_t \) is the portfolio weight on the risky asset.

Proposition 1: During retirement, the optimal consumption rule is linear in wealth

\[ c_t^r = \phi_0^r + \phi_1^r w_t^r, \text{ with } \phi_1^r = 1, \tag{4} \]

the optimal portfolio rule is

\[ a_t^r = a^r = \frac{\mu_m - r_f + 0.5\sigma_m^2}{\gamma \sigma_m^2}, \tag{5} \]
and the Euler equation for the risk-free rate is

\[ E_t[\beta e^{-\gamma \Delta c_{t+1}}] = e^{-r_{f,t+1}}. \] (6)

**Proof:** See Appendix B.

Lower-case \( c^e \) and \( w^r \) respectively represent log consumption and log wealth during retirement.\(^{11}\) The parameter \( \phi^e_r \) in equation (4) is the elasticity of consumption to wealth, which is equal to one during retirement. The constant \( \phi^e_0 \) is an endogenous, negative parameter that captures the precautionary savings motive.

The optimal decision rules after retirement are very similar to traditional life-cycle models. Specifically, based on the Euler equation for the risk-free rate in (6), we find that consumption growth does not depend on income growth because income is constant. Further, according to the expression in (5), the optimal equity share of the risky asset during retirement depends on the risk-return trade-off of the risky asset and the investor’s risk aversion, as in Merton (1969) and Samuelson (1969).

### II.D. Consumption and Portfolio Rules During Employment

Based on the optimal decisions during retirement, the investor solves for the optimal consumption and portfolio rules during employment. During employment, the consumption-portfolio problem becomes:

\[
\max_{C_t, a_t} V_t^e = \frac{C_t^{1-\gamma}}{1-\gamma} Y_t^\theta + \beta E_t \left[ \pi^e V_{t+1}^e + \pi^r V_{t+1}^r \right], \quad \text{subject to} \quad W_{t+1} = (W_t - C_t + Y_t) \left[ a_t (e^{r_{m,t+1}} - e^{r_{f,t+1}}) + e^{r_{f,t+1}} \right],
\] (7)

where \( V_t^e \) is lifetime utility while employed and \( V_t^r \) is lifetime utility when retired.

**Proposition 2:** During employment, the optimal log consumption-income difference \( (c^e_t - y_t^e) \)
$y_t$ is affine in the log wealth-income difference $(w_t^e - y_t)$,

$$c_t^e - y_t = \phi_0 + \phi_1 (w_t^e - y_t), \text{ with } 0 < \phi_1 < 1,$$

(8)

the optimal portfolio rule is

$$a^e = \frac{\mu_m - r_f + 0.5\sigma^2}{\gamma(\pi_r + \pi_e \phi_1)\sigma_m^2} - \left(1 - \phi_1 - \frac{\theta}{\gamma}\right) \frac{\pi_e \sigma_\Delta y \sigma_m}{(\pi_r + \pi_e \phi_1)\sigma_m^2} \rho_{y,m},$$

(9)

and the Euler equation for the risk-free rate between two consecutive employment periods is

$$\mathbb{E}_t[\beta e^{-\gamma \Delta c^e_{t+1} + \theta \Delta y_{t+1}}] = e^{-r_{f,t+1}}.$$

(10)

Proof: See Appendix C.

According to the consumption function in (8), the parameter $\phi_1$ is the elasticity of consumption to wealth, and $(1 - \phi_1)$ is the elasticity of consumption to income. The consumption-wealth elasticity depends on $\pi_e$ and the log-linearization constants in (25). The elasticity $\phi_1$ is always less than one, and it does not depend on the consumption-income sensitivity parameter ($\theta$) because income growth is i.i.d.

The parameter $\phi_0$ in (8) captures precautionary savings during employment. $\phi_0$ is constant because we assume that asset returns and income growth are unpredictable processes with constant volatility. In Appendix C (equation (31)), we show that consumption-income sensitivity affects $\phi_0$, which may lead to consumption that is higher, lower, or unchanged consumption relative to the traditional life-cycle model. The strength and direction of the relative change in consumption depends on the interaction between $\theta$ and the rest of the parameters.
II.E. Theoretical Predictions: Consumption-Income Sensitivity

The assumption that income is an entitlement to consumption affects the optimal consumption and portfolio decisions during employment, a period during which income is stochastic. According to the Euler equation (10) in Proposition 2, consumption growth depends on income growth. In other words, consumption growth is sensitive to current income changes. This prediction arises because income is an entitlement to consumption and thus, the marginal utility of consumption rises with income. Therefore, the license-to-spend model can rationalize the evidence that household consumption growth is excessively sensitive to current income growth.

II.F. Theoretical Predictions: Attenuation of Income Hedging

The optimal weight on the risky asset according to the license-to-spend model in equation (9) is different from models that ignore the entitlement effect. Specifically, the optimal equity share is determined by both the risk-return term, similar to Merton (1969) and Samuelson (1969), and by an income hedging term. The novel feature of our model is that the importance of income hedging depends on two confounding effects: the traditional income hedging effect driven by the consumption-income elasticity \((1 - \phi_1)\), and the preference for consumption smoothing driven by the new consumption-income sensitivity effect \((\theta/\gamma)\).

To identify the traditional income hedging motive, assume that the investor does not exhibit consumption-income sensitivity (i.e., \(\theta = 0\)). In this case, our model reduces to that of Viceira (2001), and the hedging term in equation (9) becomes

\[
-(1 - \phi_1) \frac{\pi_e \sigma \Delta y \sigma_m}{(\pi_r + \phi_1 \pi_e) \sigma_m^2} \rho_{y,m}.
\]

Because the consumption-income elasticity \((1 - \phi_1)\) does not depend on \(\theta\) and it is always less than 1, the sign of the traditional hedging term depends on the correlation between income growth and the return of the risky asset, \(\rho_{y,m}\). When \(\rho_{y,m}\) is positive, investors
have a disincentive to allocate much of their wealth to the risky asset because such an investment will magnify their total risk exposure. However, when $\rho_{y,m}$ is negative, the risky asset has income hedging benefits and investors should allocate a significant portion of their savings to the risky asset.

When consumption is sensitive to current income (i.e., $\theta > 0$), the hedging term in (9) is given by

$$-\left(1 - \phi_1 - \frac{\theta}{\gamma}ight) \frac{\pi_e \sigma_{\Delta_y} \sigma_m}{\pi_r + \phi_1 \pi_e} \sigma_m \rho_{y,m}.$$ 

In this case, if an investor exhibits strong consumption-income sensitivity, the term $1 - \phi_1 - \theta/\gamma$ is much smaller than the traditional hedging parameter $1 - \phi_1$. The model then predicts that investors with positive correlation $\rho_{y,m}$ who are also highly consumption-income sensitive should not fully hedge their income risk.

The intuition for this prediction is simple: investors with strong consumption-income sensitivity are not concerned with smoothing consumption. Consequently, they do not value risky assets as vehicles for mitigating income shocks and reducing consumption volatility. Instead, when they exhibit strong consumption-income sensitivity (i.e., $1 - \phi_1 \approx \theta/\gamma$), they want to invest a lot in assets that pay well when their income is rising, thus enhancing the consumption-entitlement effects of income. Overall, the model confirms our intuition that a strong consumption-income sensitivity confounds and attenuates the traditional income hedging motive.

In the license to spend model, marginal utility increases with income. Therefore, license to spend investors prefer assets that correlate positively (negatively) with income more (less) that the lifecycle investor. Moreover, as $\theta$ increases, the importance of the income hedging motive decreases. This mechanism is unique and complete distinguishes our model from the liquidity constraints explanation. If the license to spend parameter $\theta$ captured liquidity constraints, then as $\theta$ increases, i.e., as the price of binding liquidity constraints increases, the income hedging motive would also increase to minimize the
probability of binding liquidity constraints.\textsuperscript{12} Therefore, the liquidity explanation would have the opposite effects on income hedging to the license to spend story.

In our empirical study, we disentangle the traditional hedging component of risky asset demand from the component that is affected by the consumption-income sensitivity. To estimate our model, we decompose the optimal equity share into three components:

\[
a^e = \frac{\mu_m - r_f + 0.5\sigma^2_m}{\gamma(\pi_r + \pi_e\phi_1)\sigma^2_m} - (1 - \phi_1)\frac{\pi_e\sigma_{\Delta y}\sigma_m}{(\pi_r + \pi_e\phi_1)\sigma^2_m} \times \rho_{y,m} + \frac{\pi_e\sigma_{\Delta y}\sigma_m}{(\pi_r + \pi_e\phi_1)\sigma^2_m} \times \frac{\theta}{\gamma} \times \rho_{y,m}. \tag{11}
\]

The above decomposition breaks the optimal equity share into a risk-return ratio term, a traditional income hedging term, and an income sensitivity hedging term.

To identify each of the two hedging terms, we include two hedging control variables in our portfolio choice regressions (i.e., Probit, Tobit and Heckman regressions). The first is the correlation $\rho_{y,m}$, which captures the traditional income hedging motive and depends on the consumption-income elasticity. The second is the interaction between the consumption-income sensitivity term $\theta/\gamma$ and the correlation $\rho_{y,m}$. Our model predicts that the estimate on $\rho_{y,m}$ should be negative and the estimate on the interaction term should be positive.

\section*{III. Data and Summary Statistics}

In this section we describe the data, and present summary statistics of the main variables used in our empirical analysis.

\subsection*{III.A. Panel Study of Income Dynamics}

We use data from the Panel Study of Income Dynamics (PSID) because, to our knowledge, it is the only longitudinal survey that includes both consumption and portfolio decisions for a large sample of U.S. households.\textsuperscript{13} The long panel nature of the PSID allows

\textsuperscript{12}We would like to thank Francisco Gomes for pointing this out to us.

\textsuperscript{13}The PSID is partly supported by the National Institutes of Health under grant number R01 HD069609 and the National Science Foundation under award number 1157698.
us to estimate household-level consumption growth regressions to obtain estimates for consumption-income sensitivities.\footnote{The long panel nature of the PSID has made it a frequent data source for studies of consumption and, more recently, asset allocation (e.g., Mankiw and Zeldes (1991), Shea (1995), Dynan (2000), Brunnermeier and Nagel (2008)).}

In the first part of our empirical exercise we estimate regressions of consumption growth on income growth. For these regressions, we collect consumption and income data for all available survey years between 1978 and 2009. Our measure of consumption is total food expenditures, the sum of expenditures on food consumed at and away from home. As in many prior studies using the PSID, we treat food consumption as a proxy for total consumption (e.g., Zeldes (1989), Mankiw and Zeldes (1991), Runkle (1991), Lusardi (1996)).

We also collect income and wealth data. Our income measure is total household labor income. Wealth is measured as the household’s net worth. A large component of wealth is financial wealth that includes holdings in equities, IRA’s, and bonds, as well as checking and savings accounts.\footnote{Wealth information in the PSID is available only every five years between 1984 and 1999. Thereafter, it is available every two years.} We define stock market participants as households that hold equity directly or indirectly through IRA holdings in stocks. We also collect various demographic variables such as age, employment status, number of children, and education. Finally, we use the U.S. stock market return index and the risk-free rate from Kenneth French’s data library. Further, we deflate all asset returns, income, and consumption using the consumer price index provided by the Bureau of Labor Statistics.

We compute consumption growth and income growth to estimate consumption-income sensitivities. We also compute several household income-growth moments that we use in our portfolio choice regressions. Similar to Guiso et al. (1996) and Heaton and Lucas (2000), we define income risk as the standard deviation of income growth. To measure the traditional income hedging motive, we compute the correlation between household labor income growth and stock market returns. We compute one correlation for each household using all available data for the household, which is consistent with Vissing-
Following the literature (e.g., Runkle (1991), Vissing-Jørgensen (2002a), Angerer and Lam (2009)), we impose various sample filters. We delete household-year observations in which annual consumption growth or income growth is higher than 300% or lower than -70%, or where these quantities are missing. We also delete observations with income less than $100, and households whose income growth has a standard deviation greater than 110%.

One issue with the PSID is that surveys were administered annually prior to 1997 and only biannually after 1997. To maximize our sample size, we combine data from the annual and biannual waves. We annualize growth rates from the biannual observations by first computing the 2-year income and consumption log-growth rates, and then dividing the 2-year growth rates by 2.

Following Zeldes (1989) and Mankiw and Zeldes (1991), we interpret the PSID question on consumption as a measure of consumption during the first quarter of the survey year $t$.\textsuperscript{16} Therefore, to match the timing of consumption with the timing of the risk-free rate in the consumption growth regressions, we measure the risk free rate between the first quarter of the survey year $t$ ($Q1_t$) and the first quarter of the subsequent year $t + 1$ ($Q1_{t+1}$). We then compute the $Q1_t$-to-$Q1_{t+1}$ risk-free rate, $r_{t,t+1}$, by compounding monthly risk-free rates as in Mankiw and Zeldes (1991). After 1997, when the PSID became biannual, we compute an annualized 2-year risk-free rate by compounding the $r_{t-1,t}$ and $r_{t,t+1}$ rates, and then dividing by 2.

The existing literature also finds that the timing of the survey question regarding income is ambiguous (e.g., Zeldes (1989)). The literature typically interprets the income reported in survey year $t$ as the average income between years $t$ and $t - 1$. We adopt the same timing convention. Therefore, when we compute the correlation between income growth and market returns, we ensure that the timing of the market return follows the

\textsuperscript{16}This survey question is administered in the first quarter of the following year and refers to recent food consumption.
same convention. For example, the market return for survey year \( t \) is the average of the annual market return in year \( t \) and year \( t - 1 \).

### III.B. Summary Statistics

Summary statistics for the full sample are presented in Table I. In Table III, we present summary statistics for the portfolio choice sample. The latter sample, which is smaller than the full sample, focuses on households that do not have missing information for wealth, stock market participation, and education.

The statistics in Panel A of Table I show that consumption growth is on average about 1% whereas income growth is on average 2.5%. Both income and consumption growth are highly volatile; their standard deviation is higher than 30%. Consistent with the hypersensitivity literature, consumption and current income growth are also positively correlated; their correlation coefficient is 7.5% and it is statistically significant. The statistics in Panel B show that the average age in the full sample is 41, and only 3% of the households are retired. Finally, about 83% of households have financial assets such as savings or retirement accounts. Next, we present the regression-based evidence of consumption-income sensitivity among households in the PSID.

### IV. Consumption-Income Sensitivity Evidence

In this section, we provide evidence that PSID households, on average, do not choose to smooth consumption, and that consumption growth depends on current income growth.

#### IV.A. Empirical Specification

Following existing studies of consumer behavior (e.g., Zeldes (1989), Vissing-Jørgensen (2002b)), our estimation of consumption-income sensitivities is based on the Euler equation for the risk-free rate during two consecutive employment dates which, according to
Proposition 2, is:

$$E_t[\beta_i e^{-\gamma \Delta c_{i,t+1} + \theta \Delta y_{i,t+1}}] = e^{-r_{f,t+1}}.$$ 

In the above specification, cross-sectional heterogeneity across households is captured by differences in the rate of time preference $\beta_i$. To obtain the empirical regression, we rewrite the Euler equation by replacing the conditional expectation $E_t$ with a multiplicative error term $e^{\tilde{\epsilon}_{i,t+1}}$. Then, we take logs and solve for consumption growth $\Delta c_{i,t+1}$. The resulting expression is the following:

$$\Delta c_{i,t+1} = \frac{1}{\gamma} \log \beta_i + \frac{1}{\gamma} r_{f,t+1} + \frac{\theta}{\gamma} \Delta y_{i,t+1} + \frac{1}{\gamma} \tilde{\epsilon}_{i,t+1}.$$  \hspace{1cm} (12)

The term $\frac{1}{\gamma}$ is the elasticity of intertemporal substitution (EIS), and shows how consumption growth reacts to changes in interest rates. The parameter $\frac{\theta}{\gamma}$ captures consumption-income sensitivity, or the propensity of the household to not smooth consumption over time. Since the EIS is positive, a positive $\theta$ implies that an increase in income growth will also lead to an increase in consumption growth. When $\theta$ is zero, we obtain the Euler equation for the traditional life-cycle model.

A reduced-form specification for the Euler equation in (12) is given by

$$\Delta c_{i,t+1} = \alpha_0 + \alpha_{0,x} X_{i,t+1} + \alpha_{r_f} r_{f,t+1} + \alpha_y \Delta y_{i,t+1} + \epsilon_{i,t+1},$$  \hspace{1cm} (13)

in which $\alpha_{r_f}$ captures the EIS, $\alpha_y$ is the interaction between consumption-income sensitivity and the EIS. The rate of time preference $\log \beta_i$ is replaced by a constant, $\alpha_0$, and a vector of household control variables, $X_{i,t+1}$, that capture cross-sectional heterogeneity. These variables include age, age$^2$, number of children, as well as indicators for college education and unemployment of the household head. Since total consumption, $C_{i,t}$, is proxied by total food expenditures, $\tilde{C}_{i,t}$, at and away from home, the above specification may suffer from measurement error. However, if the measurement error in consumption
is multiplicative and independent of all other variables, $\alpha_r$ and $\alpha_y$ can be consistently estimated with ordinary least squares (Vissing-Jørgensen (2002a)).

**IV.B. Estimation of Consumption-Income Sensitivities**

We estimate the consumption growth regressions with OLS, and present the results in Table II. For the full sample case in column (2), we find that the estimate on income growth is 0.081. This estimate is statistically significant ($t$-statistic = 17.69) even in the presence of additional control variables (i.e., age, age$^2$, number of children, and indicators for college education and unemployed household head). The consumption-income sensitivity is also economically significant. A one standard deviation increase in income growth (0.32) leads to a 2.6% $(0.32 \times 0.081 \times 100)$ increase in consumption growth.

We also find that consumption growth is responsive to interest rate changes. Consistent with the results in Vissing-Jørgensen (2002a), the estimate of the EIS is positive, significant, and less than one (estimate = 0.162, $t$-statistic = 3.98). This estimate implies that a one standard deviation increase in the interest rate (0.029) leads to an increase in consumption growth of 0.47% $(0.162 \times 0.029 \times 100)$. In unreported results, we also find that based on the estimates of the consumption-income sensitivity $\alpha_y$ and the EIS, the consumption entitlement parameter $\theta$ in equation (1) is 0.466 $(0.081/0.162)$, with a $t$-statistic of 4.50.\(^{17}\) This estimate is consistent with our assumption that $\theta$ is positive and less than 1.

The significance of the consumption-income sensitivities is robust and present in various subsamples. First, the results in columns (3) and (4) indicate that consumption is sensitive to current income irrespective of the retirement status of the head-of-household. However, consistent with our model, consumption of retired households is less sensitive to income. Further, consumption-income sensitivity is also present in age-based subsamples. For example, the consumption-income sensitivity for the youngest households is 0.091 ($t$-statistic = 9.69) in column (5) and 0.041 ($t$-statistic = 5.04) for the oldest households in

\(^{17}\)The standard error for $\theta$ is calculated using the delta-method.
One potential explanation for the consumption-income sensitivity is the presence of borrowing constraints. The inability to borrow might prohibit consumers from smoothing consumption, forcing consumption expenditures to track income (e.g., Runkle (1991)). To ensure that our estimates of consumption-income sensitivity do not reflect borrowing constraints, we estimate the consumption growth regressions on subgroups of households that should not face difficulty borrowing. Drawing on Jappelli (1990), we examine households with the highest net worth (top quartile) and income (top quartile). The estimates in columns (8) and (10) of Table II show that consumption tracks current income even for the wealthiest households (estimate = 0.076, t-statistic = 5.15) and top income earners (estimate = 0.086, t-statistic = 8.55). This finding is consistent with prior evidence in the literature (e.g., Parker (2014)). Furthermore, this result is particularly important because consumption-income sensitivities are economically and statistically meaningful despite the fact that we proxy consumption with food consumption which is less sensitive to liquidity constraints.

Vissing-Jørgensen (2002a) argues that the Euler equation used in the consumption-income sensitivity literature is only valid for households that hold some form of financial assets. Following her work, we estimate the consumption growth regressions for holders and non-holders of financial assets, and report the results in columns (11) and (12) of Table II. Similar to Vissing-Jørgensen (2002a), we find that consumption growth is more responsive to interest rate changes in the subsample of financial asset holders. More importantly, the consumption-income sensitivity is significant for both groups, though it is stronger among financial asset holders.

Finally, based on our empirical results, we cannot reject the hypothesis that $\Delta y_{i,t+1}$ and $\epsilon_{i,t+1}$ in (12) are uncorrelated. This finding implies that consumption-income sensitivities are not the mechanical outcome of replacing the expectation operator with an error term in the Euler equation for the risk-free rate.

Overall, our findings from the consumption growth regressions indicate that consump-
tion is overly sensitive to income, and that this sensitivity cannot be entirely explained by borrowing constraints. Next, we use this finding, along with the predictions of our license-to-spend model, to study the relationship between consumption-income sensitivity and investment decisions.

V. Attenuation of Income Hedging

In this section, we examine the impact of consumption-income sensitivity on portfolio decisions, and provide evidence that strong consumption-income sensitivities weaken the income hedging motive in portfolio decisions. We can generalize the expression for optimal portfolio weights in (11) to allow for cross-sectional heterogeneity in consumption-income sensitivities, the EIS, and the correlation between asset returns and income growth. In this case, our model predicts that the optimal allocation to the risky asset of an employed investor is given by

\[ a^e_i = \mu_m - r_f + 0.5\sigma^2_r \frac{1}{(\pi_r + \pi_e \phi_1)\sigma^2_m} - (1 - \phi_1) \frac{\pi_e \sigma \Delta y \sigma_m}{(\pi_r + \pi_e \phi_1)\sigma^2_m} \times \rho_{y,m} + \frac{\pi_e \sigma \Delta y \sigma_m}{(\pi_r + \pi_e \phi_1)\sigma^2_m} \times \gamma_i \times \rho_{y_i,m}, \]  

or equivalently,

\[ a^e_i = b_{i,0} + b_1 \rho_{y,m} + b_2 (\alpha_{y,i} \times \rho_{y_i,m}), \]  

where the parameter \( \alpha_{y,i} \) is the consumption-income sensitivity.

The main prediction of the model is that consumption-income sensitivity attenuates the traditional hedging motive. If the correlation \( \rho_{y,m} \) is positive, the traditional hedging motive predicts that the equity share \( a^e_i \) should be low. However, when the consumption-income sensitivity effect is strong (i.e., \( \alpha_{y,i} > 0 \)), the magnitude of the traditional hedging motive on the optimal equity share, \( a^e_i \), is offset by the consumption-income sensitivity term, \( b_2 \alpha_{y,i} \). Testing this prediction is the main focus of our empirical analysis.
V.A. Household-Level Consumption-Income Sensitivities

In our portfolio analysis we estimate pooled Probit, Tobit, and two-stage Heckman regressions. The main independent variables are household-level estimates of the correlation, $\rho_{y,m}$, and of the interaction term between the correlation and the consumption-income sensitivity, $\alpha_{y,i}$.

A novel feature of our work is estimating household-level consumption-income sensitivities $\alpha_{y,i}$. Specifically, for each household we estimate a consumption growth regression,

$$\Delta c_{i,t+1} = \alpha_{i,0} + \alpha_{r,t} r_{f,t+1} + \alpha_{y,i} \Delta y_{i,t+1} + \epsilon_{i,t+1},$$

and obtain estimates of $\alpha_{y,i}$. To ensure precision in our estimates of $\alpha_{y,i}$, we focus on households that have at least 12 valid (i.e., non-missing) consumption growth observations. Also, we exclude all observations with a retired head. 18

Table III reports the average of the $\alpha_{y,i}$ estimates, which is 0.082. The average of the estimated $\alpha_{y,i}$ is reasonable because it is almost identical to the full-sample sensitivity estimate from our pooled consumption growth regressions in column (2) of Table II. In Table III, we also report summary statistics related to all of the variables we use in our portfolio choice regressions. In our sample, about 47% of respondents own stocks directly or indirectly through mutual funds and retirement accounts. On average, stockholders allocate 57% of their financial wealth to risky assets. Further, about one third of the sample is college educated, with an average age of 47.

To test the asset allocation prediction of the model, we develop the empirical counterpart of equation (15). We first proxy $\rho_{y,m}$, $\alpha_{y,m}$, and $\sigma_{\Delta y}$ with their respective estimates $\hat{\rho}_{y,m}$, $\hat{\alpha}_{y,m}$, and $\hat{\sigma}_{\Delta y}$, and add an error term $\epsilon_i$. To control for investor heterogeneity, we then include a group of control variables $Z_i$, which have been found to be significant in

18We have repeated our empirical analysis using the predictable components of income growth estimated using an AR(1) process. This alternative approach address the possibility that the estimates of the consumption-income sensitivities might be inflated if current income growth $\Delta y_{i,t+1}$ is affected by unobserved shocks that are also affecting the consumption growth term $\epsilon_{i,t+1}$. These results are very similar to the ones reported in the paper and they are available upon request.
the prior literature. These control variables are income, wealth, the standard deviation of income growth ($\sigma_{\Delta y_i}$), age, age$^2$, number of children, an unemployment indicator, and a college graduate indicator. Finally, we include a set of year indicators (year fixed effects) to capture aggregate economic conditions as well as any time-variation in the Sharpe ratio, which affects the optimal portfolio weight in (14). The reduced-form expression for the optimal equity share reads

$$a^e_{i,t} = b_t + b_{0,z}Z_{i,t} + b_1\hat{\rho}_{y,m} + b_2(\hat{\alpha}_{y,i} \times \hat{\rho}_{y,m}) + u_{i,t}. \quad (17)$$

The above relation is the basis for our empirical asset allocation regressions.

**V.B. Stock Market Participation: Probit Estimates**

We first examine whether consumption-income sensitivity affects the decision to participate in the stock market. Even if our model does not allow for participation costs, and thus all households should participate in the market, there is ample evidence that many households do not invest in risky stocks (e.g., Mankiw and Zeldes (1991), Haliasos and Bertaut (1995), Campbell (2006)). It is therefore worthwhile examining whether consumption-income sensitivity is related to the participation decision. We present results from market participation probit regressions in columns (1) and (2) of Table IV.

In the first probit regression in column (1), we include only the control variables. The estimates from this regression indicate that wealthy college graduates with high income and low income-growth volatility participate the most. In the second probit regression in column (2), we add the interaction term between the income-growth market-return correlation and the consumption-income sensitivity. We find that the estimate of the interaction term is positive and statistically significant (estimate = 0.075, $t$-statistic = 2.82). The information in the interaction term is also not related to any of the control variables because its inclusion in the probit regression does not affect their estimates and

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19Introducing time-fixed effects may affect the consistency of our age estimates. However, since age is not the main variable of interest, we opt for including both age and time-fixed effects in our regressions.
statistical significance.

The effect of the consumption-income sensitivity on participation is economically significant. For instance, consider two households with the same positive correlation $\rho_{y,m}$, equal to 0.50. However, the first household is more consumption-income sensitive. If their consumption-income sensitivities differ by one standard deviation (0.559, see Table III), then the first household will be about 2.1% ($0.075 \times 0.559 \times 0.50 \times 100$) more likely to own stocks. This effect is close to the economic effect of income risk on participation; our estimates suggest that a one-standard deviation increase in the volatility of income growth (0.115, see Table III) leads to a decrease in participation of about 2.6% ($-0.231 \times 0.115 \times 100$).

The positive effect of the $\alpha_{y,i} \times \rho_{y,m}$ term on participation is consistent with our model, which predicts that consumption-income sensitivity should attenuate the traditional income-hedging motive. However, our model does not allow for limited stock-market participation, and its predictions are largely related to the asset allocation decision, which we examine next.

V.C. Asset Allocation: Tobit Estimates

According to the license-to-spend model, consumption-income sensitivity should affect the share of financial wealth invested in risky assets. Next, we estimate Tobit regressions, and present the estimation results in columns (3) and (4) of Table IV. In these regressions the dependent variable is the percentage of financial wealth invested in stocks held directly or indirectly through retirement accounts.

The Tobit regression in column (3) includes only the control variables, while the Tobit regression in column (4) adds the consumption-income sensitivity term $\alpha_{y,i} \times \rho_{y,m}$. Consistent with previous evidence, we find that wealthy, middle-aged college graduates with high income invest the most in risky assets (e.g., Campbell (2006)). Further, we find that households with low income-growth volatility allocate more to risky assets, in line with the findings of Angerer and Lam (2009) and Betermier et al. (2012). Although
not significant, we also find that the estimate on the correlation $\rho_{y,m}$ is negative, which is consistent with the traditional income-hedging motive. In column (4), the estimate on the correlation is $-2.558$ and its $t$-statistic is $-1.02$.

The results in Table IV provide evidence that consumption-income sensitivity attenuates the traditional income-hedging motive. The estimate on the interaction term $\alpha_{y,i} \times \rho_{y,m}$ is positive and statistically significant (estimate $= 15.467$, $t$-statistic $= 3.63$). This estimate implies a strong economic effect of consumption-income sensitivity on the asset allocation decision. Consider again the two households with the same positive correlation $\rho_{y,m}$, equal to 0.50, and the first household being more consumption-income sensitive. If their consumption-income sensitivities differ by one standard deviation, 0.559 from Table III, then the first household will invest about 4.3% ($15.467 \times 0.50 \times 0.559$) more in risky assets.

Similarly, our estimates suggest that a one standard deviation increase in the income-hedging attenuation term (0.189, see Table III) leads to about a 3.0% ($15.467 \times 0.189$) increase in the allocation to stocks. This effect is economically important, and its magnitude is comparable to the effect of income risk on equity allocation. Our estimation suggests that a one standard deviation increase in the volatility of income growth (0.115) will lead to about a 4.4% ($-38.401 \times 0.115$) decrease in the equity share. This result is notable since income risk has been shown to be one of the most important determinants of equity allocation (e.g., Vissing-Jørgensen (2002b), Angerer and Lam (2009)).

V.D. Asset Allocation: Heckman Estimates

The estimates from the Tobit regressions suggest that consumption-income sensitivity weakens the traditional income hedging motive. However, the Tobit results are based on a sample that includes both stockholders and non-stockholders. To ensure that the Tobit results are not driven entirely by the participation decision, we estimate Heckman (1979) regressions that simultaneously consider the participation and asset allocation decisions.

As in Vissing-Jørgensen (2002b), we estimate a system of two equations. The first is
the participation equation estimated with data on both stockholders and non-stockholders. The first stage regression provides an estimate for the probability of participating, which is used in the second stage estimation of the equity share regression. The equity share regression is estimated using data for the stockholders only. We present the results of the joint estimation of the participation and asset allocation regressions in Table V.

Table V reports estimates from two Heckman specifications. For the first specification in columns (1) and (2), we exclude the $\alpha_{y,i} \times \rho_{y,m}$ interaction term. For the second in columns (3) and (4), we include the interaction term in order to capture the effects of consumption-income sensitivity on portfolio allocation.

V.D.1. Heckman Participation Estimates

Consistent with our previous findings, the participation regressions for both specifications in columns (1) and (3) show that wealthy college educated households with higher income and lower income growth volatility tend to participate more. Also, for the participation equation in column (3), the interaction term $\alpha_{y,i} \times \rho_{y,m}$ has a positive and statistically significant estimate (estimate $= 0.075$, $t$-statistic $= 2.81$). Therefore, households with strong consumption-income sensitivity and positive income-growth market-return correlation have a higher propensity to own stocks. For these households, the perceived costs of participation might be lower because they do not care much about consumption smoothing, and thus do not need to incur any costs to uncover the hedging potential of financial assets. However, our model does not explicitly include participation costs, and, therefore, we cannot precisely pin down the mechanism by which consumption-income sensitivity affects the participation decision.

V.D.2. Heckman Asset Allocation Estimates

The most interesting results from the Heckman system of equations are those related to the asset allocation decision. Consistent with prior evidence, older, wealthier, college educated stockholders with low income growth volatility tend to allocate more of their
wealth to risky assets. Also, stockholders with income growth that has a low correlation with market returns tend to allocate more of their wealth to risky assets. For example, in column (4), the estimate of $\rho_{y,m}$ is $-3.998$, with a $t$-statistic of $-2.55$. This significant negative estimate on the correlation term is evidence of the traditional income hedging motive.

Consistent with our model featuring consumption-income entitlements, the traditional income hedging motive is attenuated by the strength of the consumption-income sensitivity. For instance, the estimate of the interaction term $\alpha_{y,i} \times \rho_{y,m}$ in column (4) of Table V is positive ($6.641$) and statistically significant ($t$-statistic = $2.47$). This is the strongest evidence of the attenuation effect because it is based solely on households that own risky assets.

The attenuation effect is also economically significant. Consider once again the two households with the same positive correlation $\rho_{y,m}$ of $0.50$. If the consumption-income sensitivity of the first household is one standard deviation larger than that of the second ($0.559$), then the first household should allocate more of its wealth to risky assets. The Heckman estimation suggests that the first household will invest about $1.9\%$ ($6.641 \times 0.50 \times 0.559$) more in risky assets.

Similarly, the Heckman estimates suggest that a one standard deviation increase in the income-hedging attenuation term ($0.189$) leads to a $1.3\%$ ($6.641 \times 0.189$) increase in the equity share. The change of the equity share is economically significant, and its magnitude is close to the effect of income on asset allocation. Specifically, our estimates suggest that a one standard deviation increase in wealth ($0.220$) leads to about a $2.8\%$ ($12.667 \times 0.220$) increase in the proportion of wealth allocated to risky assets. Once again, this result is notable since wealth is one of the most important determinants of equity allocation (e.g., Vissing-Jørgensen (2002b), Campbell (2006)).
V.E. Measurement Error in Explanatory Variables

Overall, our findings suggest that the traditional income hedging motive is strongly attenuated in the presence of consumption-income sensitivity. A potential concern with these results is that our main variables are generated regressors. For instance, in equation (17), the consumption-income sensitivities, $\hat{\alpha}_{y,i}$, and the income growth-market return correlations, $\hat{\rho}_{y,m}$, are estimated quantities. These generated regressors could affect the consistency and statistical significance of our estimates. Therefore, we examine the impact of measurement error on our results by re-estimating the Probit, Tobit, and Heckman regressions using the block-bootstrap approach of Kunsch (1989).

We perform the bootstrap simulations by exploiting the panel structure of the PSID. Specifically, we conduct a cross-sectional bootstrap simulation in which we successively sample households with replacement. We perform one thousand bootstrap replications to compute the estimation biases and bootstrapped standard errors for the coefficient estimates in our baseline Probit, Tobit, and Heckman regressions.\(^{20}\) The estimation bias for an estimate $x$ is defined as the difference between the average of the bootstrap estimates, $\hat{E}^{(b)}[\hat{x}^{(b)}]$, and the original estimate, $\hat{x}$. The bootstrapped standard error is the standard error of the bootstrap distribution.

We report the findings from the bootstrap exercise in Table VI. For each coefficient estimate, we report the $t$-statistic calculated using bootstrapped standard errors, the estimation bias, and the $t$-statistic of the estimation bias, testing whether the bias differs significantly from zero.\(^{21}\) We report these quantities for the coefficient estimates of the Probit, Tobit, and Heckman regressions.

The results in Table VI suggest that our inference is only minimally affected by generated regressor biases. While the bootstrapped $t$-statistics of the coefficient estimates are smaller than those reported in Tables IV and V, we continue to find statistically significant attenuation of the income hedging motive due to consumption-income sensitivity.

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\(^{20}\)Our calculations are based on 2,000 bootstrap replications. Efron and Tibshirani (1993) suggest that 1,000 bootstrap replications is adequate for calculating biases and standard errors.

\(^{21}\)The $t$-statistic for the estimation bias is the bias divided by the bootstrapped standard error.
Further, in all cases, the bootstrap estimates of the biases do not differ statistically from zero. Therefore, we cannot reject the hypothesis that our original estimates are consistent. Overall, the estimation results in Table VI are similar to those reported in Tables IV and V. Thus, we conclude that potential measurement error in $\alpha_{y,i}$ and $\rho_{y,m}$ affects neither the consistency nor the significance of our baseline results.

VI. Alternative explanations

The main premise of our license-to-spend model is that investors feel happier when the time their consumption changes with their income changes. To capture this license-to-spend effect, we propose a new set of preferences in which investors derive utility from consumption as well as income. However, it is possible that the license to spend specification is theoretically isomorphic to other of alternative explanations. In this section, we provide additional robustness checks to address this concern. In the robustness tests, we add additional controls to our Probit, Tobit and Heckman regressions and present the extended regression results in Table VII. Next, we first explain the rationale behind the additional control variables and then we present the results of the extended regressions.22

VI.A. Non-linear effect of wealth and income

In our main empirical results in Tables IV and V we show that the consumption-income sensitivity hedging term ($\frac{\theta}{\gamma} \times \rho_{y,m}$) is economically and statistically significant even after controlling for income and wealth. However, there might be some non-linear effects related to income and wealth that are being captured by the interaction term $\frac{\theta}{\gamma} \times \rho_{y,m}$. Therefore, we include quadratic terms of wealth and income in the portfolio regressions in Table VII.

22Many of the alternative tests have been suggested to us by Luis Viceira, who discussed the paper at the 2014 Miami Behavioral Finance Conference, and Francisco Gomes, who discussed the paper at the 2015 AFA meetings in Boston.
VI.B. Risk aversion and endogenous labor income risk

One limitation of the PSID is that it does not provide a measure of risk aversion. To account for risk aversion we use the fact that the choice of occupation is endogenous. Exploiting this endogeneity, Ranish (2013) finds that household labor income risk is positively correlated with financial risk and risk preferences. Therefore, Ranish’s evidence suggests that labor income risk is a good proxy for risk aversion. Based on this intuition, in our main regressions in Tables IV and V, we control for risk aversions using income growth volatility. In Table VII, we include an additional control variables; the interaction between the consumption-income sensitivity measure with an indicator function that takes the value of 1 if household income growth volatility is above the median ($\theta \times \rho_{y,m} \times 1\{\sigma_{\Delta y} > \text{median}\}$). If consumption-income sensitivities are measuring risk tolerance, then this interaction term should be economically and statistically significant.

VI.C. Liquidity constraints

In Table II, we show that consumption-income sensitivities are economically and statistically significant even among the wealthiest households, suggesting that borrowing constraints cannot be the only reason we observe high consumption-income sensitivity. However, it is possible that borrowing constraints are inflating our consumption-income sensitivity estimates for highly constrained households, which in turn, inflates the significance of the income-hedging attenuation term. To account for this effect we include the consumption-sensitivity estimates as a separate control variable in the portfolio choice regressions.

We expect that adding this new control will not affect our results because according to our model, consumption-income sensitivities affect portfolio weights in a positive way through the hedging motive. In contrast, liquidity constraints should have a direct negative effect on asset allocation and stock market participation; in periods when income is low and the household is probably liquidity constrained, the household might
not participate in the market or decrease its savings in stocks to finance consumption. Therefore, the license-to-spend effect and liquidity constraints have opposite effects on portfolio decisions.

VI.D. Peer effects

Finally, the license-to-spend story might be related to models with peer or habit formation effects (e.g., Campbell and Cochrane (1999), Korniotis (2008), Gomez, Priestley, and Zapatero (2009)). The connection with peer or habit formation models comes from the fact that the utility function in these models typically take the following form:

$$U(C_t; \bar{C}_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \bar{C}_t.$$  

where $\bar{C}_t$ is the consumption of the reference group. Individual income could be a proxy for peer consumption which itself is a function of peer labor income and wealth.

In our main analysis in Tables IV and V, we use year fixed effects to account for aggregate peer effects as in the habit model of Campbell and Cochrane (1999). To account for more individualized household-level peer effects, we need the peer group of the PSID households. Unfortunately, the PSID does not have enough information to allow us to construct peer group for each household. Instead, we introduce a series of fixed effects that can potentially account for peer effects. First, we assume that investors that belong to the same occupation group should have similar peer groups. We therefore add occupation fixed effects in the regressions in Table VII. In unreported tests, we also consider household industry fixed effects and regional fixed effects based on the location of the households.\textsuperscript{23} The results with household region and industry fixed effects are very similar to the ones reported in Table VII and they are available upon request.

\textsuperscript{23}Please note that the only information in the PSID about household location is their geographical region.
VI.E. Results of extended regressions

Overall, the regressions in Table VII are based on the following extended specifications

\[
a_{i,t}^e = b_t + b_k + b_{0,t}Z_{i,t} + b_1\rho_{g,i,m} + b_2(\hat{\alpha}_{g,i} \times \hat{\rho}_{g,m}) + b_3(\hat{\alpha}_{g,i} \times \hat{\rho}_{g,m} \times 1\{\hat{\sigma}_{\Delta g_i} > \text{median}\}) + b_4\hat{\alpha}_{g,i} + u_{i,t}.
\]

in which \(b_4\) are household occupation fixed effects.

In the presence of the new control variables, we find that the marginal effects of the income hedging attenuation term \((\hat{\alpha}_{g,i} \times \hat{\rho}_{g,m})\) are statistically insignificant. In the Probit regression the marginal effect is 0.063 and its \(t\)-statistic is 1.63. In the Heckman participation regression, the marginal effect is 0.062 and its \(t\)-statistic is 1.58. This finding implies that the significance of the income hedging attenuation term in Tables IV and V is not entirely due to the license-to-spend effect. However, this finding is not that important because our license to spend model is mute about the decision to participate in the markets and it is mostly a model for asset allocation.

Moving to the asset allocation regressions, we find that the income hedging attenuation term is positive and statistically significant. Specifically, in the Tobit regressions its estimate is 14.125 and its \(t\)-statistic is 2.45. In the Heckman asset allocation regressions its estimate is 7.684. Moreover, in the Heckman asset allocation regression, the income-hedging attenuation term is the only statistically significant term \((t\)-statistic = 2.12) besides wealth and college education. The fact that the attenuation term remains significant in the extended regressions is not surprising because apart from wealth squared all the other additional controls are insignificant.

Overall, the results in Table VII show that the license to spend effect retains its explanatory power in asset allocation regressions with additional control variables. These control variables correspond to alternative explanations such as liquidity constraints, non-linear effects, endogenous labor risk and risk aversions, as well as peer effects.
VII. Summary and Conclusions

Consumption and portfolio decisions are interrelated but seldom studied together. We take a first step towards jointly examining the observed consumption and portfolio decisions of a sample of U.S. households. We find that the tendency of households to consume more when current income rises affects portfolio decisions directly. We document that consumption-income sensitivity attenuates the income hedging motive in portfolio decisions.

We formalize our empirical findings in a life-cycle model in which income is an entitlement to consume. The model is inspired by evidence from the sociology literature summarized by Akerlof (2007), and it is consistent with the behavioral economics literature on consumer behavior (e.g., Thaler (1985), Furnham and Argyle (1998), Prelec and Loewenstein (1998)). In the license-to-spend model, consumption-income entitlements undo some of the desire for consumption smoothing. Because investors are not concerned about smoothing consumption, they have a weaker incentive to hedge income fluctuations using the available menu of financial assets. Hence, the effect of the income hedging motive on their financial decisions is attenuated. Using consumption and portfolio data from the PSID, we find strong support for the attenuation effect.

We acknowledge that our theoretical model is simple in many dimensions. For example, it does not include any market participation costs or borrowing constraints. However, its simplicity allows us to analytically illustrate how consumption-income sensitivity attenuates the hedging motive. Having taken the first step to connect observed consumption decisions to portfolio decisions, we leave examination of more elaborate models of consumption entitlements for future research.
References


Panel Study of Income Dynamics (2011). Public use dataset. Produced and distributed by the Institute for Social Research, Survey Research Center, University of Michigan, Ann Arbor, MI.


TABLE I
Summary Statistics

Panel A: Consumption Growth, Income Growth and the Risk-Free Rate

<table>
<thead>
<tr>
<th></th>
<th>( \Delta c_{i,t} )</th>
<th>( \Delta y_{i,t} )</th>
<th>( r_{f,t} )</th>
<th>( \Delta c_{i,t} )</th>
<th>( \Delta y_{i,t} )</th>
<th>( r_{f,t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>1.004</td>
<td>2.479</td>
<td>2.395</td>
<td>( \Delta c_{i,t} )</td>
<td>1</td>
<td>( \Delta y_{i,t} )</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>0.000</td>
<td>1.751</td>
<td>2.337</td>
<td>( \Delta c_{i,t} )</td>
<td>( \Delta y_{i,t} )</td>
<td>( r_{f,t} )</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>37.092</td>
<td>31.966</td>
<td>2.960</td>
<td>( \Delta c_{i,t} )</td>
<td>( \Delta y_{i,t} )</td>
<td>( r_{f,t} )</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>93264</td>
<td>93264</td>
<td>93264</td>
<td>( \Delta c_{i,t} )</td>
<td>( \Delta y_{i,t} )</td>
<td>( r_{f,t} )</td>
</tr>
</tbody>
</table>

Panel B: Income, Consumption, Retirement, Age and Wealth

<table>
<thead>
<tr>
<th></th>
<th>Income</th>
<th>Consumption</th>
<th>Retired Ind.</th>
<th>Age</th>
<th>Wealth</th>
<th>Fin. Assets Ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>147315</td>
<td>2369</td>
<td>0.034</td>
<td>41</td>
<td>118435</td>
<td>0.834</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>24037</td>
<td>2060</td>
<td>0.000</td>
<td>39</td>
<td>44500</td>
<td>1.000</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>1299734</td>
<td>2445</td>
<td>0.182</td>
<td>12</td>
<td>186139</td>
<td>0.371</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>93264</td>
<td>93264</td>
<td>93264</td>
<td>93264</td>
<td>19004</td>
<td>19004</td>
</tr>
</tbody>
</table>

This table shows summary statistics for key variables in this study. \( \Delta c_{i,t} \) is consumption growth, \( \Delta y_{i,t} \) is income growth, \( r_{f,t} \) is the annual log risk-free rate. Panel A shows pooled moment and correlation estimates for the entire sample. ** shows significance at the 1% confidence level. Panel B shows summary statistics for consumption, income, retirement, age, and wealth. Consumption is measured by food at home and food out. Income is total household labor income. Retired Ind. is an indicator function depending on whether household head has retired. Wealth is household net worth, and Fin. Assets Ind. is an indicator depending on whether households hold some type of financial asset.
<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>Retirement</th>
<th>Age</th>
<th>Wealth</th>
<th>Income</th>
<th>Fin. Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2)</td>
<td>(3) (4)</td>
<td>(5) (6)</td>
<td>(7) (8)</td>
<td>(9) (10)</td>
<td>(11) (12)</td>
</tr>
<tr>
<td>$\alpha_{r_f}$</td>
<td>0.178</td>
<td>0.162</td>
<td>0.164</td>
<td>0.093</td>
<td>0.177</td>
<td>0.083</td>
</tr>
<tr>
<td></td>
<td>(4.37)</td>
<td>(3.98)</td>
<td>(3.94)</td>
<td>(0.47)</td>
<td>(1.70)</td>
<td>(1.12)</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>0.081</td>
<td>0.085</td>
<td>0.027</td>
<td>0.091</td>
<td>0.091</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>(17.69)</td>
<td>(17.76)</td>
<td>(1.66)</td>
<td>(5.14)</td>
<td>(6.14)</td>
<td>(5.15)</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.004</td>
<td>0.009</td>
<td>0.009</td>
<td>0.000</td>
<td>0.008</td>
<td>0.002</td>
</tr>
<tr>
<td>N</td>
<td>93264</td>
<td>93264</td>
<td>90061</td>
<td>3203</td>
<td>21237</td>
<td>24420</td>
</tr>
</tbody>
</table>

This table shows pooled OLS regression results for the reduced-form consumption Euler equation:

$$\Delta c_{i,t} = \alpha_0 + \alpha_{r_f} r_{f,t} + \alpha_y \Delta y_{i,t} + \alpha_{0,t} X + \epsilon_{i,t},$$

where $\Delta c_{i,t}$ is consumption growth, $r_{f,t}$ is the risk-free rate, and $\Delta y_{i,t}$ is income growth. $\alpha_{r_f}$ is the EIS, and $\alpha_y$ captures consumption-income sensitivities. The vector of control variables $X$ includes age, demeaned-age-square, female indicator for household head, number of children, unemployment indicator for household head, and an indicator for college or graduate studies. Wealth is household net worth. Financial assets include household holdings in mutual funds, IRA’s, equities, bonds, savings or checking accounts. Income is total household labor income. $t$-statistics are shown in parenthesis based on robust standard errors.
<table>
<thead>
<tr>
<th>Table III</th>
<th>Summary Statistics for Stock-Market Participation Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Participation Indicator</td>
<td>0.468</td>
</tr>
<tr>
<td>Equity Allocation</td>
<td>0.265</td>
</tr>
<tr>
<td>Participants’ Equity Allocation</td>
<td>0.567</td>
</tr>
<tr>
<td>Correlation $\hat{\rho}_{y,m}$</td>
<td>$-0.027$</td>
</tr>
<tr>
<td>Cons-Inc Sensitivity $\hat{\alpha}_{y,i}$</td>
<td>0.082</td>
</tr>
<tr>
<td>Participants’ $\hat{\alpha}_{y,i}$</td>
<td>0.081</td>
</tr>
<tr>
<td>Interaction $\hat{\alpha}<em>{y,i} \times \hat{\rho}</em>{y,m}$</td>
<td>$-0.008$</td>
</tr>
<tr>
<td>Income Growth Volatility $\hat{\sigma}_{\Delta y}$</td>
<td>0.249</td>
</tr>
<tr>
<td>Log Income $y_{i,t}$</td>
<td>10.887</td>
</tr>
<tr>
<td>Wealth</td>
<td>0.174</td>
</tr>
<tr>
<td>Age</td>
<td>46.487</td>
</tr>
<tr>
<td>Number of Children</td>
<td>0.862</td>
</tr>
<tr>
<td>Unemployment Indicator</td>
<td>0.016</td>
</tr>
<tr>
<td>College or Graduate School</td>
<td>0.337</td>
</tr>
</tbody>
</table>

This table shows summary statistics for key variables in the stock market participation model. Participation Indicator is an indicator for participating in the stock market. Equity Allocation is the fraction of wealth invested in the stock market, while Participants’ Equity Allocation is the fraction of wealth invested in the stock market conditional on having positive equity holdings. $\hat{\rho}_{y,m}$ is the correlation coefficient between income growth for household $i$ and stock market returns. $\hat{\alpha}_{y,i}$ captures consumption-income sensitivity for household $i$. $\hat{\alpha}_{y,i}$ are OLS estimates of the expression in (16) for households with more than 12 time-series observations. $\hat{\sigma}_{\Delta y}$ is income growth volatility, and $y_{i,t}$ is labor income for household $i$. Wealth is household net worth in millions of dollars.
## TABLE IV
Consumption-Income Sensitivity and Stock Market Participation:
Probit and Tobit Specifications

<table>
<thead>
<tr>
<th></th>
<th>Probit</th>
<th>Tobit</th>
<th>Probit</th>
<th>Tobit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Correlation $\hat{\rho}_{y,m}$</td>
<td>0.009</td>
<td>0.003</td>
<td>-1.375</td>
<td>-2.558</td>
</tr>
<tr>
<td></td>
<td>(0.56)</td>
<td>(0.24)</td>
<td>(-0.56)</td>
<td>(-1.02)</td>
</tr>
<tr>
<td>Interaction $\hat{\alpha}<em>{y,i} \times \hat{\rho}</em>{y,m}$</td>
<td>0.075</td>
<td>15.467</td>
<td>-36.941</td>
<td>-38.401</td>
</tr>
<tr>
<td></td>
<td>(2.82)</td>
<td>(3.63)</td>
<td>(-5.08)</td>
<td>(-5.41)</td>
</tr>
<tr>
<td>Income Growth Volatility $\hat{\sigma}_{\Delta y}$</td>
<td>-0.231</td>
<td>-0.238</td>
<td>-36.941</td>
<td>-38.401</td>
</tr>
<tr>
<td></td>
<td>(-4.94)</td>
<td>(-5.08)</td>
<td>(-5.23)</td>
<td>(-5.41)</td>
</tr>
<tr>
<td>Log Income $y_{i,t}$</td>
<td>0.093</td>
<td>0.092</td>
<td>11.210</td>
<td>11.112</td>
</tr>
<tr>
<td></td>
<td>(9.53)</td>
<td>(9.50)</td>
<td>(7.86)</td>
<td>(7.81)</td>
</tr>
<tr>
<td>Wealth</td>
<td>0.775</td>
<td>0.776</td>
<td>79.007</td>
<td>79.386</td>
</tr>
<tr>
<td></td>
<td>(20.41)</td>
<td>(20.46)</td>
<td>(21.45)</td>
<td>(21.55)</td>
</tr>
<tr>
<td>Age</td>
<td>-0.000</td>
<td>-0.000</td>
<td>0.115</td>
<td>0.116</td>
</tr>
<tr>
<td></td>
<td>(-0.70)</td>
<td>(-0.69)</td>
<td>(0.89)</td>
<td>(0.89)</td>
</tr>
<tr>
<td>Age$^2$</td>
<td>0.009</td>
<td>0.009</td>
<td>0.489</td>
<td>0.480</td>
</tr>
<tr>
<td></td>
<td>(2.41)</td>
<td>(2.39)</td>
<td>(0.80)</td>
<td>(0.78)</td>
</tr>
<tr>
<td>Number of Children</td>
<td>-0.000</td>
<td>-0.000</td>
<td>0.375</td>
<td>0.339</td>
</tr>
<tr>
<td></td>
<td>(-0.01)</td>
<td>(-0.03)</td>
<td>(0.47)</td>
<td>(0.43)</td>
</tr>
<tr>
<td>Unemployment Indicator</td>
<td>0.003</td>
<td>0.001</td>
<td>0.329</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>College or Graduate School</td>
<td>0.157</td>
<td>0.158</td>
<td>23.753</td>
<td>23.807</td>
</tr>
<tr>
<td></td>
<td>(13.94)</td>
<td>(13.99)</td>
<td>(14.22)</td>
<td>(14.27)</td>
</tr>
<tr>
<td>Constant</td>
<td>-3.718</td>
<td>-3.702</td>
<td>-154.645</td>
<td>-153.139</td>
</tr>
<tr>
<td></td>
<td>(-10.73)</td>
<td>(-10.69)</td>
<td>(-9.76)</td>
<td>(-9.69)</td>
</tr>
<tr>
<td>year FE</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Pseudo-$R^2$</td>
<td>18.99%</td>
<td>19.07%</td>
<td>3.93%</td>
<td>3.96%</td>
</tr>
<tr>
<td>N</td>
<td>7030</td>
<td>7030</td>
<td>7030</td>
<td>7030</td>
</tr>
</tbody>
</table>

This table shows Probit regressions estimates for stock market participation in columns (1) and (2). Columns (3) and (4) present estimates from Tobit regressions where the dependent variable is the portion of wealth in risky assets. For the Probit specification, we report estimates for the marginal effects. The dependent variable is an indicator function for stock market participation. $\hat{\rho}_{y,m}$ is the correlation coefficient between income growth for household $i$ and stock market returns. $\hat{\alpha}_{y,i}$ captures consumption-income sensitivity for household $i$. $\hat{\alpha}_{y,i}$ are OLS estimates of the expression in (16) for households with more than 12 time-series observations. $\hat{\sigma}_{\Delta y}$ is income growth volatility, and $y_{i,t}$ is labor income for household $i$. Wealth is household net worth in millions of dollars. $t$-statistics are shown in parenthesis and are based on robust standard errors.
### TABLE V
Consumption-Income Sensitivity and Stock Market Participation:
Two-Stage Heckman Specification (Heckman 1979)

<table>
<thead>
<tr>
<th></th>
<th>No Consumption-Income Sensitivity</th>
<th></th>
<th>Consumption-Income Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Participation</td>
<td>Equity Allocation</td>
<td>Participation</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Correlation (\hat{\rho}_{y,m})</td>
<td>0.009</td>
<td>-3.531</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
<td>(-2.27)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Interaction (\hat{\alpha}<em>{y,i} \times \hat{\rho}</em>{y,m})</td>
<td>0.075</td>
<td>6.641</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.81)</td>
<td>(2.47)</td>
<td></td>
</tr>
<tr>
<td>Income Growth Volatility (\hat{\sigma}_{\Delta y})</td>
<td>-0.233</td>
<td>-5.199</td>
<td>-0.239</td>
</tr>
<tr>
<td></td>
<td>(-4.96)</td>
<td>(-1.12)</td>
<td>(-5.10)</td>
</tr>
<tr>
<td>Log Income (y_{i,t})</td>
<td>0.093</td>
<td>-2.010</td>
<td>0.092</td>
</tr>
<tr>
<td></td>
<td>(9.52)</td>
<td>(-2.08)</td>
<td>(9.49)</td>
</tr>
<tr>
<td>Wealth</td>
<td>0.775</td>
<td>12.493</td>
<td>0.776</td>
</tr>
<tr>
<td></td>
<td>(20.60)</td>
<td>(4.30)</td>
<td>(20.65)</td>
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<tr>
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<td>-0.000</td>
<td>0.120</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>(-0.69)</td>
<td>(1.23)</td>
<td>(-0.69)</td>
</tr>
<tr>
<td>Age(^2)</td>
<td>0.009</td>
<td>-0.919</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(2.41)</td>
<td>(-1.94)</td>
<td>(2.39)</td>
</tr>
<tr>
<td>Number of Children</td>
<td>0.000</td>
<td>0.199</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.38)</td>
<td>(-0.02)</td>
</tr>
<tr>
<td>Unemployment Indicator</td>
<td>0.003</td>
<td>3.366</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.94)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>College or Graduate School</td>
<td>0.158</td>
<td>4.716</td>
<td>0.158</td>
</tr>
<tr>
<td></td>
<td>(13.96)</td>
<td>(3.79)</td>
<td>(14.00)</td>
</tr>
<tr>
<td>Constant</td>
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<td>56.489</td>
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<td></td>
<td>(-10.73)</td>
<td>(4.61)</td>
<td>(-10.69)</td>
</tr>
<tr>
<td>year FE</td>
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<td>yes</td>
<td>yes</td>
</tr>
<tr>
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<td>3291</td>
<td>7030</td>
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</tbody>
</table>

This table shows two-stage Heckman (Heckman 1979) regressions for stock market participation based on the expression for optimal portfolio weights in (17). In the selection equation, the dependent variable is an indicator function for stock market participation, while in the equity allocation equation, the dependent variable is the percentage of total wealth allocated to risky assets. For the participation equation, we report estimates for the marginal effects. \(\hat{\rho}_{y,m}\) is the correlation coefficient between income growth for household \(i\) and stock market returns. \(\hat{\alpha}_{y,i}\) captures consumption-income sensitivity for household \(i\). \(\hat{\alpha}_{y,i}\) are OLS estimates of the expression in (16) for households with more than 12 time-series observations. \(\hat{\sigma}_{\Delta y}\) is income growth volatility, and \(y_{i,t}\) is labor income for household \(i\). Wealth is household net worth in millions of dollars. \(t\)-statistics are shown in parenthesis and are based robust standard errors.
This table shows bootstrapped results for the stock participation and allocation regressions in Tables IV and V. Estimates are the initial estimates. Bias is the bootstrapped estimate for the bias which is defined as the difference between the average estimate of the bootstrap distribution and the initial estimate. For the Probit specification and the Participation equation, we report estimates for the marginal effects, while the corresponding biases refer to parameter estimates. t-statistics are shown in parentheses, and are based on block-bootstrapped standard errors with resampling at the household-level.
Alternative Explanations

<table>
<thead>
<tr>
<th></th>
<th>Probit</th>
<th>Tobit</th>
<th>Participation</th>
<th>Equity Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Correlation $\hat{\rho}_{y,m}$</td>
<td>0.000</td>
<td>-1.859</td>
<td>0.000</td>
<td>-3.799</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(-0.53)</td>
<td>(0.03)</td>
<td>(-1.79)</td>
</tr>
<tr>
<td>(2) Cons-Inc sensitivity $\hat{\alpha}_{y,i}$</td>
<td>0.001</td>
<td>-0.362</td>
<td>0.001</td>
<td>-0.628</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(-0.19)</td>
<td>(0.13)</td>
<td>(-0.51)</td>
</tr>
<tr>
<td>(3) Interaction $\hat{\alpha}<em>{y,i} \times \hat{\rho}</em>{y,m}$</td>
<td>0.063</td>
<td>14.125</td>
<td>0.062</td>
<td>7.684</td>
</tr>
<tr>
<td></td>
<td>(1.63)</td>
<td>(2.43)</td>
<td>(1.58)</td>
<td>(2.12)</td>
</tr>
<tr>
<td>(4) Income Growth Volatility $\hat{\sigma}_{\Delta y}$</td>
<td>-0.200</td>
<td>-30.706</td>
<td>-0.203</td>
<td>-6.221</td>
</tr>
<tr>
<td></td>
<td>(-3.17)</td>
<td>(-3.13)</td>
<td>(-3.21)</td>
<td>(-1.00)</td>
</tr>
<tr>
<td>(5) Interaction $\hat{\alpha}<em>{y,i} \times \hat{\rho}</em>{y,m} \times 1{\hat{\sigma}_{\Delta y} &gt; median}$</td>
<td>0.082</td>
<td>8.900</td>
<td>0.085</td>
<td>-3.457</td>
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<tr>
<td></td>
<td>(0.86)</td>
<td>(0.58)</td>
<td>(0.88)</td>
<td>(-0.39)</td>
</tr>
<tr>
<td>(6) Log Income $y_{i,t}$</td>
<td>0.077</td>
<td>10.773</td>
<td>0.077</td>
<td>-2.123</td>
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<td></td>
<td>(5.32)</td>
<td>(3.61)</td>
<td>(5.29)</td>
<td>(-1.08)</td>
</tr>
<tr>
<td>(7) Log Income$^2$ $y_{i,t}^2$</td>
<td>-0.004</td>
<td>-2.022</td>
<td>-0.004</td>
<td>0.077</td>
</tr>
<tr>
<td></td>
<td>(-0.59)</td>
<td>(-1.71)</td>
<td>(-0.63)</td>
<td>(0.11)</td>
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<tr>
<td>(8) Wealth</td>
<td>1.078</td>
<td>152.122</td>
<td>1.084</td>
<td>23.832</td>
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<tr>
<td></td>
<td>(17.56)</td>
<td>(16.42)</td>
<td>(17.65)</td>
<td>(3.98)</td>
</tr>
<tr>
<td>(9) Wealth$^2$</td>
<td>-0.680</td>
<td>-110.130</td>
<td>-0.687</td>
<td>-10.877</td>
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<td>(-8.41)</td>
<td>(-10.13)</td>
<td>(-8.52)</td>
<td>(-1.74)</td>
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<tr>
<td>(10) Age</td>
<td>-0.001</td>
<td>-0.112</td>
<td>-0.001</td>
<td>0.106</td>
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<td>(-1.39)</td>
<td>(-0.69)</td>
<td>(-1.42)</td>
<td>(0.93)</td>
</tr>
<tr>
<td>(11) Age$^2$</td>
<td>0.010</td>
<td>0.865</td>
<td>0.010</td>
<td>-0.917</td>
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<td></td>
<td>(2.07)</td>
<td>(1.19)</td>
<td>(2.06)</td>
<td>(-1.62)</td>
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<tr>
<td>(12) Number of Children</td>
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<td>0.244</td>
<td>-0.000</td>
<td>0.208</td>
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<td>(-0.05)</td>
<td>(0.25)</td>
<td>(-0.02)</td>
<td>(0.34)</td>
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<tr>
<td>(13) Unemployment Indicator</td>
<td>0.002</td>
<td>0.420</td>
<td>0.003</td>
<td>2.441</td>
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<td>(0.05)</td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.67)</td>
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<tr>
<td>(14) College or Graduate School</td>
<td>0.128</td>
<td>18.772</td>
<td>0.129</td>
<td>4.940</td>
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<td></td>
<td>(7.56)</td>
<td>(7.36)</td>
<td>(7.62)</td>
<td>(3.05)</td>
</tr>
<tr>
<td>(15) Constant</td>
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<td>-150.612</td>
<td>-3.281</td>
<td>50.576</td>
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<tr>
<td></td>
<td>(-6.40)</td>
<td>(-4.75)</td>
<td>(-6.36)</td>
<td>(2.29)</td>
</tr>
</tbody>
</table>

HH occupation FE yes yes yes yes
year FE yes yes yes yes
N 7030 7030 7030 3291

This table shows Probit, Tobit, and two-stage Heckman (Heckman 1979) regressions for stock market participation based on the expression for optimal portfolio weights in (18). In the probit and two-stage Heckman selection equations, the dependent variable is an indicator function for stock market participation, while in the tobit and the equity allocation equations, the dependent variable is the percentage of total wealth allocated to risky assets. For the Probit model and the participation equation, we report estimates for the marginal effects. $\hat{\rho}_{y,m}$ is the correlation coefficient between income growth for household $i$ and stock market returns. $\hat{\alpha}_{y,i}$ captures consumption-income sensitivity for household $i$. $\hat{\alpha}_{y,i}$ are OLS estimates of the expression in (16) for households with more than 12 time-series observations. $\hat{\sigma}_{\Delta y}$ is income growth volatility, and $y_{i,t}$ is labor income for household $i$. Wealth is household net worth in millions of dollars. HH occupation FE are household occupation fixed effects. $t$-statistics are shown in parenthesis and are based on block-bootstrapped standard errors with resampling at the household-level.
Appendix For Online Publication

A Properties of Consumption-Income Utility Function

In this section, we show that the utility function is increasing and concave in \( C_t \), and that the Inada conditions hold. The first derivative of the utility function in (1) with respect to \( C_t \) is

\[
\frac{\partial U(C_t; Y_t)}{\partial C_t} = C_t^{1-\gamma} Y_t^\theta. \tag{19}
\]

Because \( 0 < Y_t < +\infty \) and \( \gamma > 1 \), the marginal utility of consumption is bounded:

\[
\lim_{C_t \downarrow 0} \frac{\partial U(C_t; Y_t)}{\partial C_t} = +\infty,
\]

and

\[
\lim_{C_t \uparrow +\infty} \frac{\partial U(C_t; Y_t)}{\partial C_t} = 0.
\]

The above conditions imply that individuals always consume part of their income, and that consumption is strictly positive. Therefore, the derivative in (19) is also positive, and the inverse consumption ratio \( Y_t/C_t \) is well defined.

Moreover, marginal utility is an increasing function of \( Y_t \) because the cross-derivative,

\[
\frac{\partial^2 U(C_t; Y_t)}{\partial C_t \partial Y_t} = \theta C_t^{-\gamma} Y_t^{\theta-1},
\]

is positive for \( \theta > 0 \). Finally, the second derivative with respect to \( C_t \) is

\[
\frac{\partial^2 U(C_t; Y_t)}{\partial C_t^2} = -\gamma C_t^{-\gamma-1} Y_t^\theta,
\]

which is negative for \( \gamma > 0 \).
B Proof of Proposition 1

We prove Proposition 1 by first log-linearizing the budget constraint and the Euler equations of the problem. Then, we follow the method of undetermined coefficients to solve for the optimal consumption and portfolio rules. Specifically, we make a guess about the optimal consumption and equity share policy functions, we verify our guesses, and derive the optimal portfolio rules.

First, consider the budget constraint

\[ W_{t+1} = (W_t - C_t + \bar{Y})R_{p,t+1}. \]

in which \( R_{p,t+1} \) are portfolio returns. After dividing both sides by \( W_t + \bar{Y} \), the log-linearized version of the budget constraint around \( \bar{c} \) and \( \bar{w} \) reads

\[ w_{t+1} - \tilde{\lambda}_0^r - \tilde{\lambda}_1^r w_t = \tilde{\kappa}_0^r - \tilde{\kappa}_1^r c_t + \tilde{\kappa}_2^r w_t + r_{p,t+1}, \]

where \( \tilde{\kappa}_0^r = \log(1 - \frac{e^{\bar{w}}}{e^{\bar{w} + \bar{Y}}}) + \tilde{\kappa}_1^r \bar{c} - \tilde{\kappa}_2^r \bar{w}, \tilde{\kappa}_1^r = \frac{1}{1 - e^{\bar{w} + \bar{Y}}} \frac{e^{\bar{w}}}{e^{\bar{w} + \bar{Y}}}, \) and \( \tilde{\kappa}_2^r = \frac{1}{1 - e^{\bar{w} + \bar{Y}}} \frac{e^{\bar{w}}}{e^{\bar{w} + \bar{Y}}} \). Also, \( \tilde{\lambda}_0^r = \log(e^{\bar{w}} + \bar{Y}) - \tilde{\lambda}_1^r \bar{w} \) and \( \tilde{\lambda}_1^r = \frac{e^{\bar{w}}}{e^{\bar{w} + \bar{Y}}} \).

Setting \( e^{\bar{w}} \) to be much larger than \( \bar{Y} \), then \( \tilde{\lambda}_1^r = \frac{e^{\bar{w}}}{e^{\bar{w} + \bar{Y}}} \approx 1, \tilde{\kappa}_1^r \approx \tilde{\kappa}_2^r, \) and the log-linearized budget constraint simplifies to

\[ w_{t+1} - w_t = \kappa_0^r - \kappa_1^r (c_t - w_t) + r_{p,t+1}, \] \hspace{1cm} (20)

where \( \kappa_0^r = \log(1 - \frac{e^{\bar{w}}}{e^{\bar{w} + \bar{Y}}}) + \kappa_1^r (\bar{c} - \bar{w}) + \log(e^{\bar{w}} + \bar{Y}) - \bar{w}, \) and \( \kappa_1^r = \frac{1}{1 - e^{\bar{w} + \bar{Y}}} \frac{e^{\bar{w}}}{e^{\bar{w} + \bar{Y}}} \).

Next, we log-linearize the expression for the excess portfolio returns, which are defined as

\[ e^{r_m,t+1} - e^{r_f,t+1} = a_t (e^{r_m,t+1} - e^{r_f,t+1}). \] \hspace{1cm} (21)

In order to simplify the expression (21), we use a log-normal approximation for the excess portfolio returns \( (e^{r_p,t+1} - e^{r_f,t+1}) \) and the excess returns for the risky asset \( (e^{r_m,t+1} - e^{r_f,t+1}) \)
to obtain
\[ r_{p,t+1} - r_{f,t+1} = a_t (r_{m,t+1} - r_{f,t+1}) + 0.5a_t (1 - a_t) \sigma_m^2. \]

Next, we log-linearize the Euler equations of the three assets during retirement. From the problem in (3), we know that the Euler equations are:

\[ E_t [\beta e^{-\gamma \Delta c_{t+1}^r + r_{t+1}^i}] = 1, \quad i \in \{p, m, f\}. \]

The log-linearised Euler equations for portfolio \( p \) and the risky asset \( m \) are:

\[
\begin{align*}
log \beta - \gamma E_t [\Delta c_{t+1}^r] + E_t [r_{i,t+1}] + 0.5 \text{Var}_t [r_{i,t+1} - \gamma \Delta c_{t+1}^r] &= 0, \quad i \in \{p, m\},
\end{align*}
\]

while the log-linearised Euler equation for the risk-free asset is:

\[
\begin{align*}
log \beta - \gamma E_t [\Delta c_{t+1}^r] + r_f + 0.5 \text{Var}_t [-\gamma \Delta c_{t+1}^r] &= 0.
\end{align*}
\]

We use the log-linearized budget constraint and the log-linearized Euler equations to solve for the optimal consumption and optimal equity share. To find the optimal consumption and portfolio policies, we follow a guess and verify approach where we make a guess about the optimal policy rule, and then use the log-linearized budget constraint and Euler equations to verify our guess.

Suppose that the optimal consumption policy is given by \( c_t^r = \phi_0^r + w_t^r \), and that the optimal portfolio rule is constant across time, i.e., \( a_t^r = a^r \). Our goal is to pin down the parameter \( \phi_0 \), so that our guesses satisfy the log-linearized budget constraint and the Euler equations. Under our two guesses, consumption growth is related to wealth growth, i.e., \( \Delta c_{t+1}^r = \Delta w_{t+1}^r \), and hence \( E_t [\Delta c_{t+1}^r] = E_t [\Delta w_{t+1}^r] \). Using the log-linearized budget constraint in (20) and our guess for the optimal portfolio rule \( (a_t^r = a^r) \), we also obtain that

\[
E_t [\Delta c_{t+1}^r] = a^r (\mu_m - r_f) + r_f + 0.5a^r (1 - a^r) \sigma_m^2 + \kappa_0^r - \kappa_1^r \phi_0^r.
\]
On the other hand, the Euler equation for portfolio returns implies that
\[
E_t[\Delta c^r_{t+1}] = \frac{1}{\gamma} \left[ \log \beta + E_t[r^r_{p,t+1}] + 0.5 Var_t[r^r_{p,t+1} - \gamma \Delta c^r_{t+1}] \right].
\]

Since \( \Delta c^r_{t+1} = \Delta w^r_{t+1} \), we have that
\[
Var_t[r^r_{p,t+1} - \gamma \Delta c^r_{t+1}] = Var_t[r^r_{p,t+1} - \gamma \Delta w^r_{t+1}].
\]

Using the log-linearized budget constraint in (20) and the guess that \( a^r_t = a^r \), we can write the right-hand side of the above expression as:
\[
Var_t[r^r_{p,t+1} - \gamma \Delta w^r_{t+1}] = (1 - \gamma)^2 (a^r)^2 \sigma_m^2.
\]

Thus, the Euler equation implies that expected consumption growth is equal to
\[
E_t[\Delta c^r_{t+1}] = \frac{1}{\gamma} \left[ \log \beta + a^r(\mu_m - r_f) + r_f + 0.5 a^r (1 - a^r) \sigma_m^2 + 0.5 (1 - \gamma)^2 (a^r)^2 \sigma_m^2 \right]. \tag{23}
\]

Equalizing the two expressions in (22) and (23), we obtain the solution for \( \phi^r_0 \)
\[
\phi^r_0 = \frac{a^r(\mu_m - r_f) + r_f + 0.5 a^r (1 - a^r) \sigma_m^2 + \kappa_0^r}{\kappa_1^r} \\
- \frac{1}{\kappa_1^r \gamma} \left[ \log \beta + a^r(\mu_m - r_f) + r_f + 0.5 a^r (1 - a^r) \sigma_m^2 + 0.5 (1 - \gamma)^2 (a^r)^2 \sigma_m^2 \right].
\]

To derive the optimal portfolio weight \( a^r \), subtract the log-linearized Euler equation for the risk-free asset from the Euler equation for the risky asset:
\[
\mu_m - r_f + 0.5 \sigma_m^2 = \gamma Cov_t(r_{m,t+1}, \Delta c^r_{t+1}).
\]

Using our guess for optimal consumption rules \( c^r_t = \phi^r_0 + w^r_t \), the log-linearized budget
constraint in (20), and our guess for the optimal portfolio rule \((a_t^* = a^*)\), we obtain that

\[
\mu_m - r_f + 0.5\sigma_m^2 = \gamma \text{Cov}_t(r_{m,t+1}, a^* r_{m,t+1}) \iff a^* = \frac{\mu_m - r_f + 0.5\sigma_m^2}{\gamma \sigma_m^2}.
\]

C Proof of Proposition 2

We prove Proposition 2 is a similar manner as Proposition 1. That is, we log-linearize the budget constraint and the Euler equations. Then, we follow a guess-and-verify approach to derive the optimal consumption and equity share rules.

First, we log-linearize the pre-retirement budget constraint:

\[
W_{t+1} = (W_t - C_t + Y_t)R_{p,t+1}.
\] (24)

Dividing both sides of the budget constraint by \(Y_{t+1}\), the log-linearized version of the budget constraint around \(\overline{w} - \overline{y}\) and \(\overline{c} - \overline{y}\) is

\[
w_{t+1} - y_{t+1} = \kappa_0 + \kappa_1 (w_t - y_t) - \kappa_2 (c_t - y_t) - \Delta y_{t+1} + r_{p,t+1},
\] (25)

where

\[
\begin{align*}
\kappa_1 &= \frac{e^{\overline{w} - \overline{y}}}{1 + e^{\overline{w} - \overline{y}} - e^{\overline{c} - \overline{y}}}, \\
\kappa_2 &= \frac{e^{\overline{c} - \overline{y}}}{1 + e^{\overline{w} - \overline{y}} - e^{\overline{c} - \overline{y}}} \quad \text{and} \\
\kappa_0 &= \log[1 + e^{\overline{w} - \overline{y}} - e^{\overline{c} - \overline{y}}] - \kappa_1 (\overline{w} - \overline{y}) + \kappa_2 (\overline{c} - \overline{y}).
\end{align*}
\] (26)

Next, we simplify the Euler equations. From problem (7), the pre-retirement Euler equations are

\[
\pi_e E_t[\beta e^{-\gamma \Delta c_{t+1} + \Delta y_{t+1}} + r_{i,t+1}] + \pi_r E_t[\beta e^{-\gamma \Delta y_{t+1} + (\bar{y} - y_t)} + r_{i,t+1}] = 1, \quad i \in \{p, m, f\}.
\]

Because we assumed that pension income \(\bar{y}\) is constant and equal to the last pre-retirement
income payment, the difference \( \bar{y} - y_t \) in the above expression is zero. Therefore,

\[
\pi_e \mathbb{E}_t [\beta e^{-\gamma \Delta c_{t+1}^e + \theta \Delta y_{t+1} + r_{t+1}}] + \pi_r \mathbb{E}_t [\beta e^{-\gamma \Delta c_{t+1}^r + r_{t+1}}] = 1, \ i \in \{p, m, f\}. \tag{27}
\]

Based on equation (27) above, the second-order Taylor approximations of the Euler equations for the portfolio \( p \) and the risky asset \( m \) are

\[
\pi_e \left\{ \log \beta - \gamma \mathbb{E}_t [\Delta c_{t+1}^e] + \theta \mathbb{E}_t [\Delta y_{t+1}] + \mathbb{E}_t [r_{t+1}] + 0.5 \text{Var}_t [-\gamma \Delta c_{t+1}^e + \theta \Delta y_{t+1} + r_{t+1}] \right\} +
\pi_r \left\{ \log \beta - \gamma \mathbb{E}_t [\Delta c_{t+1}^r] + \mathbb{E}_t [r_{t+1}] + 0.5 \text{Var}_t [-\gamma \Delta c_{t+1}^r + r_{t+1}] \right\} = 0, \ i \in \{p, m\}.
\]

Also, the log-linearised Euler equation for the risk-free asset yields

\[
\pi_e \left\{ \log \beta - \gamma \mathbb{E}_t [\Delta c_{t+1}^e] + \theta \mathbb{E}_t [\Delta y_{t+1}] + r_f + 0.5 \text{Var}_t [-\gamma \Delta c_{t+1}^e + \theta \Delta y_{t+1}] \right\} +
\pi_r \left\{ \log \beta - \gamma \mathbb{E}_t [\Delta c_{t+1}^r] + r_f + 0.5 \text{Var}_t [-\gamma \Delta c_{t+1}^r] \right\} = 0.
\]

Using the identity: \( \Delta c_{t+1} = (c_{t+1} - y_t) - (c_t - y_t) + \Delta y_t, \ s \in \{e, r\} \), the Euler equation for portfolio returns \( p \) becomes

\[
\log \beta - \gamma \sum_{s = e, r} \pi_s \mathbb{E}_t [c_{t+1}^s - y_{t+1}] + \gamma (c_t^s - y_t) - \gamma \mathbb{E}_t [\Delta y_{t+1}] + \theta \pi_e \mathbb{E}_t [\Delta y_{t+1}] + \mathbb{E}_t [r_{p, t+1}] +
0.5 \pi_e \text{Var}_t [-\gamma (c_{t+1}^e - y_{t+1}) + \gamma (c_t^e - y_t) - \gamma \Delta y_{t+1} + \theta \Delta y_{t+1} + r_{p, t+1}] +
0.5 \pi_r \text{Var}_t [-\gamma (c_{t+1}^r - y_{t+1}) + \gamma (c_t^r - y_t) - \gamma \Delta y_{t+1} + r_{p, t+1}] = 0.
\]

Finally, we use the guess-and-verify method to obtain the optimal policy rules. In particular, we guess that portfolio weights are constant, i.e., \( a_t^e = a^e \), and that the log consumption-income ratio is linear in wealth and income

\[
c_{t+1}^e - y_{t+1} = \phi_0 + \phi_1 (w_{t+1}^e - y_{t+1}).
\]
We can also rewrite the optimal consumption policy during retirement as
\[
c^*_{t+1} - y_{t+1} = \phi^*_0 + \phi^*_1 (w^e_{t+1} - y_{t+1}),
\]
with \( \phi^*_1 = 1 \). Note that even if our investor retires at time \( t + 1 \), equation (24) still describes the evolution of her wealth from time \( t \) to time \( t + 1 \). This is why we use \( w^e_{t+1} \) in (28) rather than \( w^r_{t+1} \). Plugging the above guesses into the Euler equation for portfolio \( p \), we get
\[
0 = \log\beta - \gamma \left[ \pi_e \left( \phi_0 + \phi_1 \mathbb{E}_t[w^e_{t+1} - y_{t+1}] \right) + \pi_r \left( \phi^*_0 + \mathbb{E}_t[w^e_{t+1} - y_{t+1}] \right) \right] + \\
\gamma [\phi_0 + \phi_1 (w^e_t - y_t)] - \gamma \mathbb{E}_t[\Delta y_{t+1}] + \theta \pi_e \mathbb{E}_t[\Delta y_{t+1}] + \mathbb{E}_t[r^e_{p,t+1}] + \\
0.5 \pi_e Var_t \left[ \gamma ((\phi_0 + \phi_1 (w^e_{t+1} - y_{t+1})) - [\phi_0 + \phi_1 (w^e_t - y_t)] + \Delta y_{t+1} - \theta \Delta y_{t+1} - r^e_{p,t+1}] + \\
0.5 \pi_e Var_t \left[ \gamma ((\phi^*_0 + (w^e_{t+1} - y_{t+1})) - [\phi^*_0 + \phi^*_1 (w^e_t - y_t)] + \Delta y_{t+1} - r^e_{p,t+1}] \right].
\]

Using the log-linearized budget constraint in (25),
\[
0 = \log\beta - \gamma \left[ \pi_e \left( \phi_0 + \phi_1 \mathbb{E}_t[\kappa_0 + \kappa_1 (w^e_t - y_t) - \Delta y_{t+1} + r^e_{p,t+1}] \right) + \\
\pi_r \left( \phi^*_0 + \mathbb{E}_t[\kappa_0 + \kappa_1 (w^e_t - y_t) - \Delta y_{t+1} + r^e_{p,t+1}] \right) \right] + \\
\gamma [\phi_0 + \phi_1 (w^e_t - y_t)] - \gamma \mathbb{E}_t[\Delta y_{t+1}] + \theta \pi_e \mathbb{E}_t[\Delta y_{t+1}] + \mathbb{E}_t[r^e_{p,t+1}] + \\
0.5 \pi_e Var_t \left[ \gamma [\phi_0 + \phi_1 [\kappa_0 + \kappa_1 (w^e_t - y_t) - \Delta y_{t+1} + r^e_{p,t+1}]] + \\
\gamma \Delta y_{t+1} - \theta \Delta y_{t+1} - r^e_{p,t+1}] + \\
0.5 \pi_e Var_t \left[ [\phi^*_0 + \kappa_0 + \kappa_1 (w^e_t - y_t) - \Delta y_{t+1} + r^e_{p,t+1}]] + \gamma \Delta y_{t+1} - r^e_{p,t+1}] \right].
\]
Once more, our guess for the optimal consumption-income ratio implies that

\[
0 = \log \beta - \gamma \left[ \pi_e \left( \phi_0 + \phi_1 \left( \kappa_0 + \kappa_1 (w^e_t - y_t) - \kappa_2 (\phi_0 + \phi_1 (w^e_t - y_t)) \right) - \mathbf{E}_t[\Delta y_{t+1}] + \mathbf{E}_t[r^e_{p,t+1}] \right) \right] + \pi_r \left( \phi_0^r + \kappa_0 + \kappa_1 (w^e_t - y_t) - \kappa_2 (\phi_0 + \phi_1 (w^e_t - y_t)) - \mathbf{E}_t[\Delta y_{t+1}] + \mathbf{E}_t[r^e_{p,t+1}] \right) + \\
\gamma (\phi_0 + \phi_1 (w^e_t - y_t)) - \gamma \mathbf{E}_t[\Delta y_{t+1}] + \theta \mathbf{E}_t[\Delta y_{t+1}] + \mathbf{E}_t[r^e_{p,t+1}] + \\
0.5 \pi_e \text{Var}_t \left[ \gamma (\phi_0 + \phi_1 \left( \kappa_0 + \kappa_1 (w^e_t - y_t) - \kappa_2 (\phi_0 + \phi_1 (w^e_t - y_t)) \right) - \Delta y_{t+1} + r^e_{p,t+1} \right] + \\
0.5 \pi_r \text{Var}_t \left[ \gamma (\phi_0 + \kappa_1 (w^e_t - y_t) - \kappa_2 (\phi_0 + \phi_1 (w^e_t - y_t)) - \Delta y_{t+1} + r^e_{p,t+1} \right] + \gamma \Delta y_{t+1} - r^e_{p,t+1}. \]
\]

Since \( a^e_t = a^e \) and \( \Delta y_{t+1} \) is an i.i.d. process, the Euler equation becomes

\[
\log \beta - \gamma \left[ \pi_e \left( \phi_0 + \phi_1 \left( \kappa_0 + \kappa_1 (w^e_t - y_t) - \kappa_2 (\phi_0 + \phi_1 (w^e_t - y_t)) \right) - \mu_y + \alpha^e (\mu_m - r_f) + r_f + 0.5 \alpha^e (1 - a^e) \sigma_m^2 \right) \right] + \\
\pi_r \left( \phi_0^r + \kappa_0 + \kappa_1 (w^e_t - y_t) - \kappa_2 (\phi_0 + \phi_1 (w^e_t - y_t)) - \mu_y + \alpha^e (\mu_m - r_f) + r_f + 0.5 \alpha^e (1 - a^e) \sigma_m^2 \right) + \\
\gamma (\phi_0 + \phi_1 (w^e_t - y_t)) - \gamma \mu_y + \theta \pi_e \mu_y + \alpha^e (\mu_m - r_f) + r_f + 0.5 \alpha^e (1 - a^e) \sigma_m^2 + \\
0.5 \pi_e \text{Var}_t \left[ \gamma (\phi_1 [\Delta y_{t+1} + r^e_{p,t+1}]) \right] + \gamma \Delta y_{t+1} - \theta \Delta y_{t+1} + r^e_{p,t+1} + \\
0.5 \pi_r \text{Var}_t \left[ \gamma [\Delta y_{t+1} + r^e_{p,t+1}] + \gamma \Delta y_{t+1} - r^e_{p,t+1} \right] = 0. \tag{29}
\]

Collecting \( w^e_t - y_t \) terms, the following equation in \( \phi_1 \) must hold

\[
-\gamma \pi_e \phi_1 \kappa_1 + \gamma \pi_e \kappa_2 (\phi_1)^2 - \gamma \pi_r \kappa_1 + \gamma \pi_r \kappa_2 \phi_1 + \gamma \phi_1 = 0.
\]

Both solutions for the quadratic equation above are real and have opposite signs because the constant term \((-\frac{\pi \kappa_1}{\pi \kappa_2})\) is negative. Since \( \phi_1 \) is the elasticity of consumption to wealth, it has to be positive. Therefore, we choose the positive solution which makes intuitive and economic
sense, and conclude that

$$
\phi_1 = \frac{(\pi_e \kappa_1 - \pi_r \kappa_2 - 1) + \sqrt{(1 + \pi_r \kappa_2 - \pi_e \kappa_1)^2 + 4 \pi_e \kappa_2 \pi_r \kappa_1}}{2 \pi_r \kappa_2}.
$$

Finally, \( \phi_0 \) depends on all the remaining constant terms in (29)

$$
\log \beta - \gamma \pi_e \phi_0 - \gamma \pi_e \phi_1 \kappa_0 + \gamma \pi_e \phi_1 [\mu_y + a^e (\mu_m - r_f)] + r_f + 0.5 a^e (1 - a^e) \sigma_m^2
$$

$$
-\gamma \pi_r \phi_0 - \gamma \pi_r \kappa_0 + \gamma \pi_r [\mu_y + a^e (\mu_m - r_f)] + r_f + 0.5 a^e (1 - a^e) \sigma_m^2 + \gamma \phi_0 - \gamma \mu_y + \theta \pi_e \mu_y
$$

$$
+a^e (\mu_m - r_f) + r_f + 0.5 a^e (1 - a^e) \sigma_m^2 + 0.5 \pi_e (1 - \phi_1)^2 (a^e)^2 \sigma_m^2 + 0.5 \pi_r \gamma^2 (1 - \phi_1 - \frac{\theta}{\gamma})^2 \sigma_{\Delta y}^2
$$

$$
-\pi_e (1 - \gamma \phi_1) a^e \gamma (1 - \phi_1 - \frac{\theta}{\gamma}) \rho_{\mu, \gamma} \sigma_{\Delta y} + 0.5 \pi_r (1 - \gamma)^2 (a^e)^2 \sigma_m^2 = 0.
$$

Returning to optimal portfolio weights, subtract the log-linearized Euler equation for the risk-free asset from the log-linearized Euler equation for the risky asset to get

$$
\mu_m - r_f + 0.5 \sigma_m^2 = \gamma \left[ \pi_e \text{Cov}_t \left( r_{m,t+1}, \Delta c^e_{t+1} \right) + \pi_r \text{Cov}_t \left( r_{m,t+1}, \Delta c^r_{t+1} \right) \right] - \pi_e \text{Cov}_t \left( r_{m,t+1}, \Delta y_{t+1} \right).
$$

Using the identity: \( c^s_{t+1} - c^r_t = (c^s_{t+1} - y_{t+1}) - (c^r_t - y_t) + \Delta y_{t+1} \), \( s \in \{ r, e \} \), we obtain that

$$
\mu_m - r_f + 0.5 \sigma_m^2 = \gamma \left[ \pi_e \text{Cov}_t \left( r_{m,t+1}, (c^e_{t+1} - y_{t+1}) - (c^e_t - y_t) + \Delta y_{t+1} \right) + \pi_r \text{Cov}_t \left( r_{m,t+1}, (c^r_{t+1} - y_{t+1}) - (c^r_t - y_t) + \Delta y_{t+1} \right) \right] - \pi_e \text{Cov}_t \left( r_{m,t+1}, \Delta y_{t+1} \right).
$$

Our guess about optimal consumption policy implies that

$$
\mu_m - r_f + 0.5 \sigma_m^2 = \gamma \left[ \pi_e \text{Cov}_t \left( r_{m,t+1}, \phi_1 (w^e_{t+1} - y_{t+1}) + \Delta y_{t+1} \right) + \pi_r \text{Cov}_t \left( r_{m,t+1}, \phi_1^r (w^r_{t+1} - y_{t+1}) + \Delta y_{t+1} \right) \right] - \pi_e \text{Cov}_t \left( r_{m,t+1}, \Delta y_{t+1} \right).
$$

Using the log-linearized budget constraint in (25), we obtain that

$$
\mu_m - r_f + 0.5 \sigma_m^2 = \gamma \left[ \pi_e \text{Cov}_t \left( r_{m,t+1}, \phi_1 (r_{p,t+1} - \Delta y_{t+1}) + \Delta y_{t+1} \right) + \pi_r \text{Cov}_t \left( r_{m,t+1}, \phi_1^r (r_{p,t+1} - \Delta y_{t+1}) + \Delta y_{t+1} \right) \right] - \pi_e \text{Cov}_t \left( r_{m,t+1}, \Delta y_{t+1} \right).
$$
Since $a_t^e = a^e$ and $\phi_1^e = 1$, the previous expression becomes

$$
\mu_m - r_f + 0.5\sigma^2_m = \gamma [\pi_e \text{Cov}(r_{m,t+1}, \phi_1(a^e r_{m,t+1} - \Delta y_{t+1}) + \Delta y_{t+1}) + 
\pi_e \text{Cov}(r_{m,t+1}, (a^e r_{m,t+1} - \Delta y_{t+1}) + \Delta y_{t+1})] - \theta \pi_e \text{Cov}(r_{m,t+1}, \Delta y_{t+1}).
$$

We can solve the above expression with respect to $a^e$, and find that

$$
a^e = \frac{\mu_m - r_f + 0.5\sigma^2_m}{\gamma (\pi_r + \pi_e \phi_1) \sigma^2_m} - \left(1 - \phi_1 - \frac{\theta}{\gamma}\right) \frac{\pi_e \rho_y \sigma \Delta y \sigma_m}{(\pi_r + \pi_e \phi_1) \sigma^2_m}.
$$

Lastly, we show that $\phi_1 < 1$ such that the term $1 - \phi_1$ in the portfolio hedging motive is positive. The proof is by contradiction. Suppose that $\phi_1 \geq 1$, then

$$
\phi_1 = \frac{(\pi_e \kappa_1 - \pi_r \kappa_2 - 1) + \sqrt{(1 + \pi_r \kappa_2 - \pi_e \kappa_1)^2 + 4\pi_e \kappa_2 \pi_r \kappa_1}}{2\pi_e \kappa_2} \geq 1 \iff
\sqrt{(\pi_e \kappa_1 - \pi_r \kappa_2 - 1)^2 + 4\pi_e \kappa_2 \pi_r \kappa_1} \geq 2\pi_e \kappa_2 + (1 + \pi_r \kappa_2 - \pi_e \kappa_1) \iff
(1 + \pi_r \kappa_2 - \pi_e \kappa_1)^2 + 4\pi_e \kappa_2 \pi_r \kappa_1 \geq 4\pi_e \kappa_2^2 + (1 + \pi_r \kappa_2 - \pi_e \kappa_1)^2 + 4\pi_e \kappa_2 (1 + \pi_r \kappa_2 - \pi_e \kappa_1) \iff
\pi_r \kappa_1 \geq \pi_e \kappa_2 + (1 + \pi_r \kappa_2 - \pi_e \kappa_1) \iff 0 \geq 1 + \kappa_2 - \kappa_1.
$$

The last inequality is false, since the definition of $\kappa_1$ and $\kappa_2$ in (26) implies that $1 + \kappa_2 - \kappa_1 > 0$. 

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